ON TIPS NOTES

CLASS X (MATHEMATICS)

CHAPTER 1 : Real Numbers

- **Fundamentals :**
  1. A non-negative integer 'p' is said to be divided by an integer 'q' if there exists an integer 'd' such that :
  $$p = qd.$$
  2. ± 1 divides every non-zero integer.
  3. 0 does not divide any integer.
- **Euclid's Division Lemma :** Let a and b be any two positive integers, then there exists unique integers q and r such that : $a = bq + r$, where, $0 \le r < b$
- **Euclid's Division Algorithm :** Let a and b be any two positive integers such that $a > b$ and 'q' and 'r' as quotient and remainder, respectively

  Take a as dividend and b as divisor

  $\therefore$ $a = bq + r$, where, $0 \le r < b$

  then every common divisor of a and b is a common divisor of b and r.

  Finding HCF of two positive integers using Euclid's Division Algorithm :

  Step 1 : Apply Euclid's lemma to a and b : $a = bq_1 + r_1$.

  Step 2 : If $r_1 = 0$, then HCF $= b$.

  Step 3 : If $r_1 \neq 0$, then again apply Euclid's lemma : $b = q_1 r_1 + r_2$

  Step 4 : If $r_2 = 0$, then HCF $= r_1$.

  Step 5 : If $r_2 \neq 0$, then again apply Euclid's lemma till remainder $r_n = 0$, then the divisor is the HCF.
- **Fundamental Theorem of Arithmetic :**

  Every composite number can be expressed as a product of primes, and this factorisation is unique except for the order in which the prime factors occur.

  Important Theorems :
  1. Let p be a prime number and a be a positive integer. If p divides a^2, then p divides a.
  2. Consider two positive integers a and b, then LCM $\times$ HCF $= a \times b$.
  3. Let x be a rational number whose decimal expansion terminates. Then we can express x in the form $\frac{p}{q}$, where p and q are co-prime and the prime Factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers.
  4. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.
  5. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which is non-terminating repeating.

HCF and LCM :
1. HCF (a, b) : Product of the smallest power of each common prime factor in the numbers.
2. LCM (a, b) : Product of the greatest power of each prime factor, involved in the numbers.
3. HCF $(a, b) \times$ LCM $(a, b) = a \times b$, for any two positive integers a and b.

CHAPTER 2 : Polynomials

- **Fundamentals :**

  An algebraic equation of the form,

  $$p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots\dots\dots + a_{n-1}x + a_n$$

  is called polynomial, provided it has no negative exponent for any variable.

  where, $a_0, a_1, a_2, \dots\dots a_{n-1}, a_n$ are constants (real numbers); $a_0 \neq 0$.

- **Degree of polynomial :** n is called the degree (highest power of variable x). If $n = 1$ then polynomial is called linear polynomial.

  General form : $ax + b = 0$ where, $a \neq 0$

  If $n = 2$ then polynomial is called quadratic polynomial.

  General form: $ax^2 + bx + c = 0$ where, $a \neq 0$

  If $n = 3$ then polynomial is called cubic polynomial.

  General form : $ax^3 + bx^2 + cx + d = 0$ where, $a \neq 0$

- **Zeroes of Polynomial :** For polynomial $p(x)$, the value of x for which $p(x) = 0$, is called zero(es) of polynomial.

  Linear Polynomial can have atmost 1 root.

  Quadratic Polynomial can have atmost 2 roots.

  Cubic Polynomial can have atmost 3 roots.

- Relationship between zeroes and coefficient of polynomial :

  - Zero of linear polynomial $ax + b = 0$ is given by $x = \frac{-b}{a} = \frac{-(\text{Constant term})}{(\text{Coefficient of } x)}$
  - If α and β are zeroes of the quadratic polynomial $ax^2 + bx + c = 0$, then

    Sum of zeroes $$\alpha + \beta = \frac{-b}{a} = -\frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$$

    Product of zeroes $$\alpha\beta = \frac{c}{a} = \frac{(\text{Constant term})}{(\text{Coefficient of } x^2)}$$

  - If a, b and g are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d = 0$, then

    Sum of zeroes $$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(\text{Coefficient of } x^2)}{(\text{Coefficient of } x^3)}$$

    $$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^3)}$$

    Product of zeroes $$\alpha\beta\gamma = \frac{-d}{a} = \frac{-(\text{Constant term})}{(\text{Coefficient of } x^3)}$$

- Division algorithm for polynomials : If $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then we can find $q(x)$ and $r(x)$ such that

  $$p(x) = g(x) \times q(x) + r(x),$$

  In simple words : Dividend = Divisor × Quotient + Remainder

  Tips : 1. Graph of linear equation is a straight line, while graph of quadratic equation is a parabola.

  2. Degree of polynomial = maximum number of zeroes of polynomial.

  3. If remainder $r(x) = 0$, then $g(x)$ is a factor of $p(x)$.

  4. To form quadratic polynomial if sum and product of zeroes are given

  $$p(x) = x^2 - (\text{Sum of zeroes})\, x + (\text{Product of zeroes})$$

CHAPTER 3 : Pair of Linear Equations in Two Variables

- **Fundamentals :**

1. The general form of linear equation in one variable is : $ax + b = 0, (a \neq 0)$.

2. The general form of linear equation in two variables is: $ax + by + c = 0$.

where, a, b, c are real coefficients; ($a^2 + b^2 \neq 0$ *i.e.*, a and b are not both zero).

3. Every linear equation gives a straight line graph. Every point lying on this line is a solution of linear equation.

4. Two linear equations which are in the same two variables x and y simultaneously are called pair of linear equations in two variables.

5. The general form of pair of linear equation in two variables is :

$$a_1x + b_1y + c_1 = 0$$

and $$a_2x + b_2y + c_2 = 0$$

where, $a_1, b_1, c_1, a_2, b_2, c_2$ are all real coefficients.

6. A pair of values of x and y satisfying each one of the equations is called solution of the system.

- **Graphical and Algebraic Interpretation :**

Pair of Linear Equations	Algebraic Condition	Graphical Interpretation	Algebraic Interpretation	Consistency
$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique solution	Consistent
	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions	Dependent Consistent
	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	Inconsistent

- **Methods of Solving Pair of Linear Equations :**

(A) Graphical method

(B) Algebraic methods

(i) Substitution method

(ii) Elimination method

(iii) Cross-Multiplication method

Graphical Method :

Step 1 : Plot the both linear equations on the same graph.

Step 2 : Find the intersecting point on graph, if the lines are intersecting.

Step 3 : Intersecting point is the required solution.

Step 4 : In step 2, if the lines are coincident, then there are infinitely many solutions each point on the line being a solution.

Step 5 : In step 2, if the lines are parallel, then the pair of equations has no solution.

Substitution Method :

Step 1 : From one equation, find the value of one variable in terms of other variable.

Step 2 : Substitute the value of variable obtained in step 1 in the other equation, you will get the equation in one variable.

Step 3 : Solve the equation in one variable and find the value of variable.

Step 4 : Substitute the value of the variable so obtained in step 3 in any equation, you will get equation in unknown variable.

Step 5 : Solve this equation in one variable and find the value of this variable.

Elimination Method :

Step 1 : First multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.

Step 2 : Then add or subtract one equation from other so that one variable gets eliminated and resultant equation will become an equation in one variable.

Step 3 : Solve the equation in one variable and find the value of the variable.

Step 4 : Substitute the value of the variable so obtained in step 3 in any equation, you will get equation in unknown variable.

Step 5 : Solve this equation in one variable and find the value of this variable now.

Cross Multiplication Method : For a pair of linear equations,

$$a_1x + b_1y + c_1 = 0$$

and $a_2x + b_2y + c_2 = 0$, make an array as below :

$$\frac{x}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{y}{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

From this, you will get :

$$\boxed{x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}}, a_1b_2 - a_2b_1 \neq 0$$

Equations Reducible to a pair of linear equations in two variable : There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so that they are reduced to a pair of linear equations.

CHAPTER 4 : Quadratic Equations

- **Fundamentals :**
  1. General form of quadratic equation is : $ax^2 + bx + c = 0$, Where $a \neq 0$.
  2. a is co-efficient of x^2, b is co-efficient of x, c is called constant term.
  3. Equation of the form $ax^2 + c = 0$ is called pure quadratic equation.
  4. The value of variable satisfying equation is called root of that equation.
  5. Quadratic equation has atmost two roots.
- **Methods of Solving Quadratic Equations :**
  - **(a)** Factorisation Method (Splitting the Middle Term)
  - **(b)** Completing the Square Method
  - **(c)** Quadratic Formula Method (Sridharacharya Formula)

Factorisation Method :

Step 1 : Resolve the equation in factor using splitting the middle term method, *i.e,* : $ax^2 + bx + c = (Ax + B)(Cx + D)$

Step 2 : Put both factors equal to zero,

$Ax + B = 0$ and $Cx + D = 0$

Therefore : $x = -\frac{B}{A}$ and $x = -\frac{D}{C}$ are two roots.

Completing the Square Method :

Step 1 : Let the equation be : $ax^2 + bx + c = 0$

Step 2 : Make the co-efficient of x^2 unity by dividing whole equation by a, you will get :

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Step 3 : Shift the constant term on RHS, you will get :

$$x^2 + \frac{bx}{a} = -\frac{c}{a}$$

Step 4 : Add the square of half of the co-efficient of x on the both sides of the equation :

$$x^2 + 2\left[\frac{b}{2a}\right]x + \left[\frac{b}{2a}\right]^2 = -\frac{c}{a} + \left[\frac{b}{2a}\right]^2$$

Step 5 : Write LHS as the perfect square and simplify RHS :

$$\left[x + \frac{b}{2a}\right]^2 = \frac{b^2 - 4ac}{4a^2}$$

Step 6 : Take square root on both sides :

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

Step 7 : Evaluate the values of x by shifting the constant term on RHS.

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$\Rightarrow$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- **Quadratic Formula Method :**

  Direct formula to calculate the roots is given as :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  Proof : Use steps given in completing the square method.

  Discriminant : It is denoted by 'D' and given by :

$$D = b^2 - 4ac$$

- **Nature of Roots of Quadratic equation :**

  First of all find D.

  - If $D > 0$, roots are real and unequal.
  - If $D = 0$, roots are real and equal.
  - If $D < 0$, roots are imaginary.

CHAPTER 5 : Arithmetic Progressions

- **Sequence :** An arrangement of numbers which has a pattern, which can suggest the successor of every number in the arrangement.
- **Examples of Arithmetic Progressions :**

  3, 5, 7, 9, 11..........

  – 8, – 5, – 2, 1, 4, 7........

  6, 1, – 4, – 9, – 14.........

  $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ .... Yes it is AP, because it can be written as below :

  $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2}$ ....

  $3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, 3 + 4\sqrt{2}, 3 + 5\sqrt{2}$.............
- **Arithmetic Progression (AP) :** It is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

  This fixed number is called common difference, denoted by 'd'. It can be +ve, – ve.

  So general form of an AP is given by :

  $a, a + d, a + 2d, a + 3d, a + 4d, \ldots\ldots, a + (n - 1)d.$

  where, a = first term, d = common difference and $a + (n - 1)d = n^{th}$ term.

  If t_1, t_2, t_3 .... be the Ist, IInd, IIIrd...... terms of an AP.

  then, $t_2 - t_1 = d$

  $t_3 - t_2 = d$

  $t_4 - t_3 = d$ and so on.....................

  Hence : It can be written :

  $t_2 - t_1 = t_3 - t_2$

  $2t_2 = t_1 + t_3$

  Conclusion : If three numbers a, b and c are in AP then :

  $2b = a + c.$
- **Important Formulae :**

  n^{th} term of an AP $T_n = a + (n - 1)d$

  Sum of the n terms of an AP is : $S_n = \frac{n}{2}[2a + (n - 1)d]$

  Also : $S_n = \frac{n}{2}(a + l)$

  where, l = last term, *i.e.*, $l = a + (n - 1)d.$

  1. n^{th} term from the end of an AP : $(l - (n - 1)d)$

  2. $t_n = (S_n - S_{n-1})$
- **Tips :**

  1. To an AP if we (i) add (ii) subtract (iii) multiply or (iv) divide each term by the same number, the resulting sequence would always be an AP.

  2. Whenever you be asked to take three numbers which are in AP, always take :

  $a - d, a, a + d.$

  3. Whenever you be asked to take four numbers which are in AP, always take :

  $(a - 3d), (a - d), (a + d), (a + 3d)$

  4. Whenever you be asked to take five numbers which are in AP, always take :

  $(a - 2d), (a - d), (a), (a + d), (a + 2d)$

- **Proof Of "Sum of n terms of an AP"**

We know that general form of an AP is given by :

$a, a + d, a + 2d, a + 3d, a + 4d, \ldots\ldots\ldots\ldots, a + (n-1)d.$

$\Rightarrow S_n = a + (a + d) + (a + 2d) + (a + 3d) + \ldots\ldots\ldots\ldots + [a + (n-2)d] + [a + (n-1)d]$ .....(i)

Now write the above equation in reverse order :

$\Rightarrow S_n = [a + (n-1)d] + [a + (n-2)d] + \ldots\ldots\ldots\ldots\ldots + (a + 3d) + (a + 2d) + (a + d) + a$ ....(ii)

Adding the corresponding terms of eq (i) & (ii), we get

$\Rightarrow 2S_n = [2a + (n-1)d] + [2a + (n-1)d] + [2a + (n-1)d] \ldots\ldots\ldots\ldots\ldots\ldots [2a + (n-1)d]$

$\Rightarrow 2S_n = n[2a + (n-1)d]$

$\Rightarrow S_n = \frac{n}{2}[2a + (n-1)d]$

Also, it can be written as : $S_n = \frac{n}{2}[a + a + (n-1)d] = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_n = \frac{n}{2}[a + l]$ [where, last term, $l = a + (n-1)d$] **Hence Proved.**

- **Some important key points :**

1. We know $a_n = a + (n-1)d$

$a_n = a + nd - d$

$a_n = (a - d) + nd$

i.e., Linear equation denotes general term where

(i) co-efficient of n is common difference 'd'

(ii) constant term is $(a - d)$.

2. We know $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = na + \frac{n}{2}(n-1)d$

$S_n = na + n^2\frac{d}{2} - n\frac{d}{2}$

$S_n = n\left(a - \frac{d}{2}\right) + n^2\frac{d}{2}$

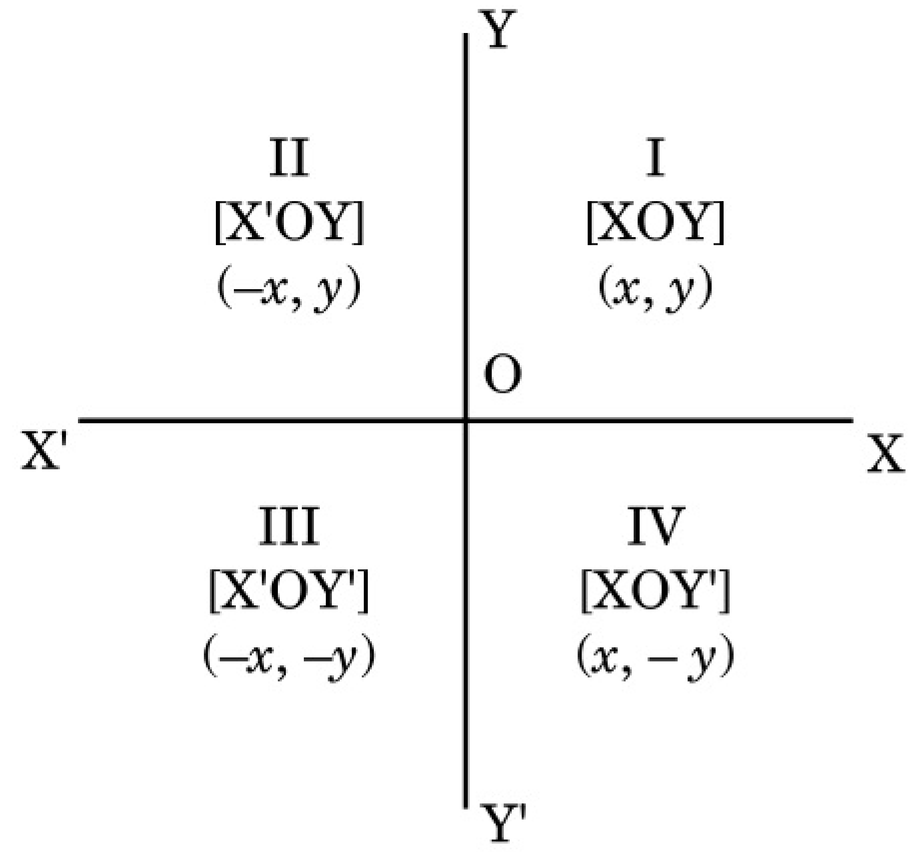
i.e., Quadratic equation denotes sum to n terms where

(i) Co-efficient of n^2 is $\frac{d}{2}$

(ii) Co-efficient of n is $\left[a - \frac{d}{2}\right]$

CHAPTER 6 : Lines (In Two Dimensions)

- **Fundamentals :**
  - **(i)** Distance of any point from the y-axis is called x co-ordinate or **abscissa**.
  - **(ii)** Distance of any point from the x-axis is called y co-ordinate or **ordinate**.
  - **(iii)** Origin : (0, 0)
  - **(iv)** Point on x-axis : $(x, 0)$.
  - **(v)** Point on y-axis : $(0, y)$
  - **(vi)** There are four quadrants in a co-ordinate plane :



- **Distance Formula :**

  Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Corollary : Distance of point $A(x, y)$ from origin is $\sqrt{x^2 + y^2}$

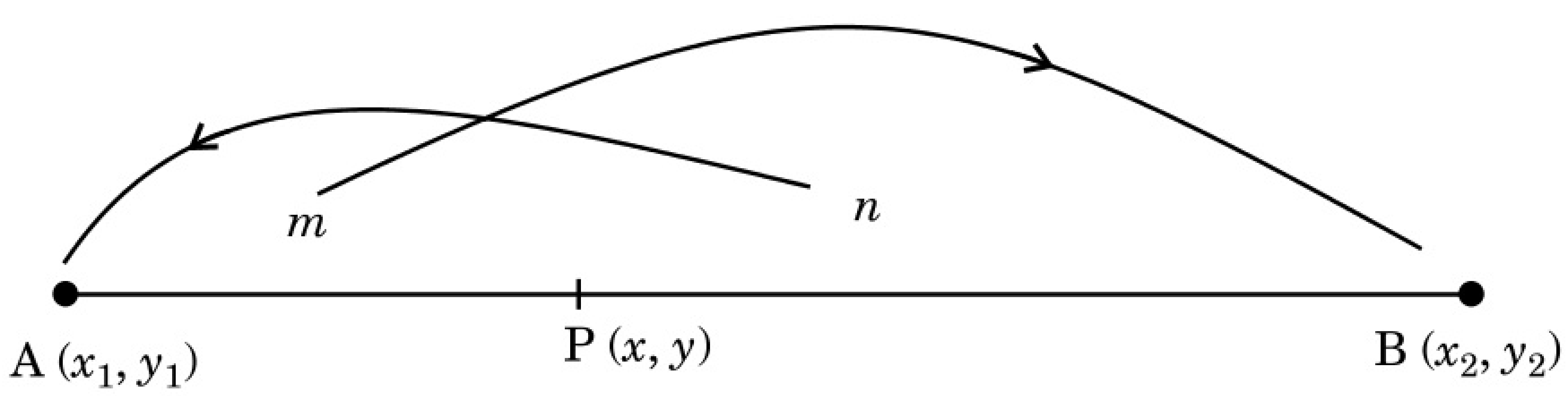
- **Tips :** Co-ordinates will form :
  1. Rhombus, if all the four sides are equal.
  2. Square, if all the four sides and diagonals are equal.
  3. Parallelogram, if opposite sides are equal.
  4. Rectangle, if opposite sides and diagonals are equal.
  5. Right, triangle, if it follows Pythagoras theorem.
  6. Collinearity condition. [A, B, C are collinear if $AB + BC = AC$]

- **Section Formula :**

  Co-ordinates of the point $P(x, y)$, dividing the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ are given by :

$$x = \frac{mx_2 + nx_1}{m+n}, \; y = \frac{my_2 + ny_1}{m+n}$$

- **How to remember the section formula ?**



- **Corollary :** If $P(x, y)$ is the mid-point, therefore $m : n = 1 : 1$

$$x = \frac{x_2 + x_1}{2}, \; y = \frac{y_2 + y_1}{2}$$

- **Tips :** If the ratio in which P divides AB is not given then we take assumed ratio as $k : 1$.

- **Centroid Formula** : Co-ordinates of the centroid $G(x, y)$ of triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) are given by

$$x = \frac{x_1 + x_2 + x_3}{3},\ y = \frac{y_1 + y_2 + y_3}{3}$$

Area Of Triangle : Area of triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by :

$$A = \frac{1}{2}\ |[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]|$$

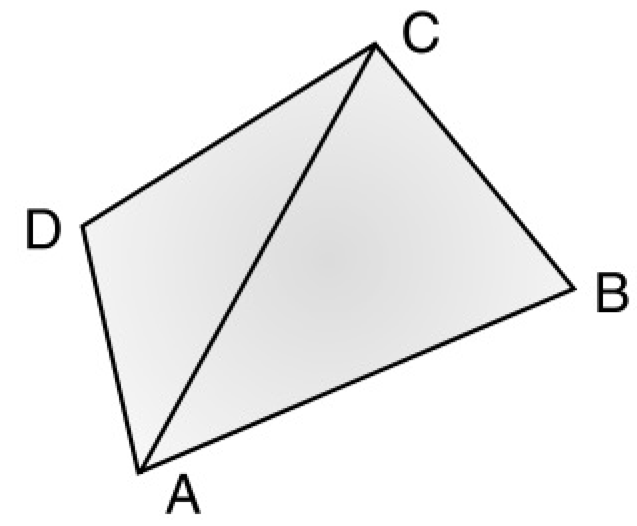
Note : | | (modulus) sign means many times answer is negative, but, | | sign consider it to be positive.

➢ **Tips** :

1. To prove three points to be collinear, Area of Triangle formed by these three points = 0 *i. e.*,

$$|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]| = 0$$

2. Area of Quadrilateral $ABCD$ = Area of ΔABC + Area of ΔACD.



CHAPTER 7 : Triangles

- **Fundamentals :**

  Similar figures : Two figures of same shape are said to be similar if :

  1. Their corresponding angles are equal.
  2. Their corresponding sides are proportional.

  Examples :

  1. All circles
  2. All squares
  3. All equilateral triangles
  4. All congruent triangles.

- **Statements of Theorems**

  1. Basic proportionality Theorem : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

  2. Converse of Basic Proportionality Theorem : If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.

  3. Ratio of the areas of two similar triangles : The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

  4. Pythagoras Theorem : In a right, triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

  5. Converse of Pythagoras theorem : In a triangle, if the square of one side is equal to sum of the squares of the other two sides, the angle opposite to the first side is a right angle.

  6. If a perpendicular is drawn from the vertex of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.

- **Criterion for similarity of two triangles**

  (i) *SSS* **Similarity :** If the corresponding sides of two triangles are proportional, then triangles are similar.

  (ii) *AAA* **Similarity :** If the corresponding angles of two triangles are equal, then triangles are similar.

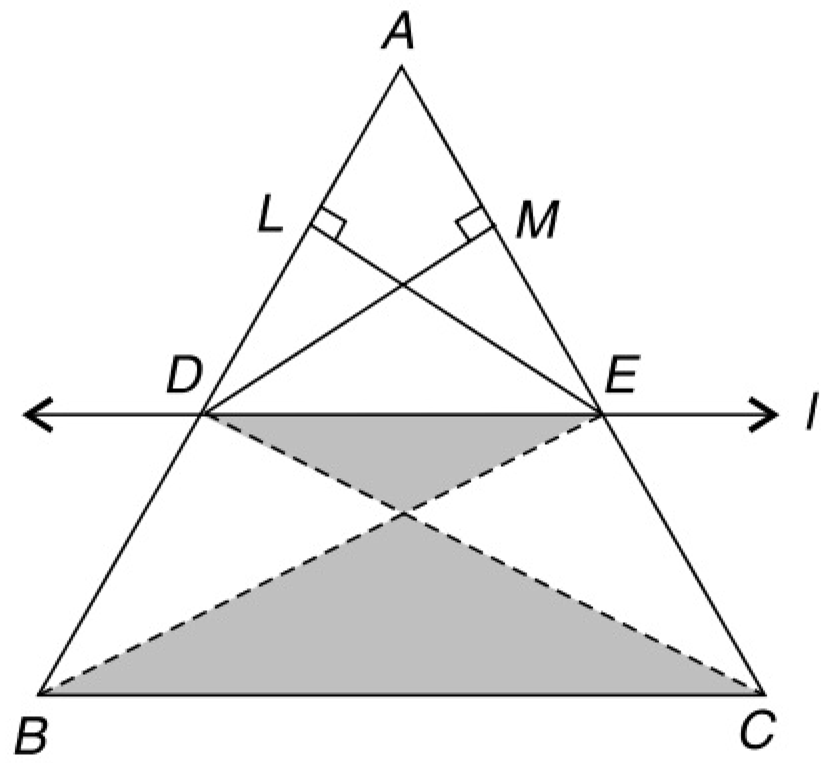
  (iii) *SAS* **Similarity :** If the pair of corresponding sides of two triangles are proportional and the included angles are equal, then triangles are similar.

Proof of Theorems :

1. **Basic Proportionality Theorem (Thales Theorem) :**

Statement : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Given : A ΔABC and line 'l' parallel to BC intersect AB at D and AC at E.



To Prove : $$\frac{AD}{DB} = \frac{AE}{EC}$$

Construction : Join BE and CD. Draw $EL \perp$ to AB and $DM \perp AC$.

Proof : We know that areas of the triangles on the same base and between same parallel lines are equal, hence we have :

$$\text{area}(\Delta BDE) = \text{area}(\Delta CDE) \qquad ...(i)$$

Now, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2} \times AD \times EL}{\frac{1}{2} \times DB \times EL} = \frac{AD}{DB} \qquad ...(ii)$$

Again, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \qquad ...(iii)$$

Put value form (i) in (ii), we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{AD}{DB} \qquad ...(iv)$$

On comparing equation (ii) and (iii), we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$ **Hence Proved.**

Corollary :

(i) $\frac{AB}{DB} = \frac{AC}{EC}$ (ii) $\frac{DB}{AD} = \frac{EC}{AE}$

(iii) $\frac{AB}{AD} = \frac{AC}{AE}$ (iv) $\frac{DB}{AB} = \frac{EC}{AC}$

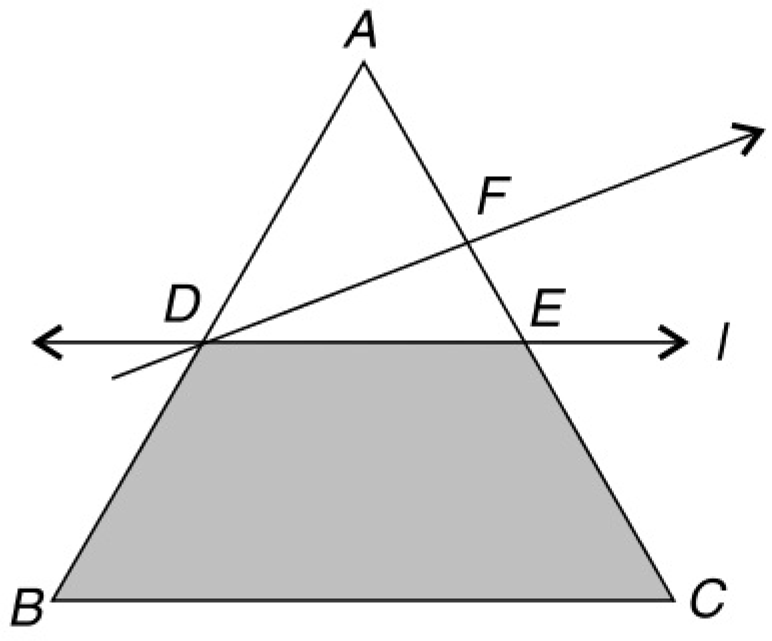
(v) $\frac{AD}{AB} = \frac{AE}{AC}$

2. Converse of Basic Proportionality :

Statement : If a line divides any two sides of a triangle in the same ratio, the line must be parallel to the third side.

Given : A ΔABC and line '*l*' intersecting the sides *AB* at *D* and *AC* at *E* such that :

$$\frac{AD}{DB} = \frac{AE}{EC}$$



To Prove : $l \parallel BC$.

Proof : Let us suppose that the line *l* is not parallel to *BC*.

Then through *D*, there must be any other line which must be parallel to *BC*.

Let $DF \parallel BC$, such that $E \neq F$.

Since, $DF \parallel BC$ (by supposition)

$$\frac{AD}{DB} = \frac{AF}{FC} \quad \text{(Basic Proportionality Theorem) ...(i)}$$

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{(Given) ...(ii)}$$

Comparing (i) and (ii), we get

$$\frac{AF}{FC} = \frac{AE}{EC}$$

Adding 1 to both sides, we get

$$\frac{AF}{FC}+1 = \frac{AE}{EC}+1$$

$$\Rightarrow \quad \frac{AF+FC}{FC} = \frac{AE+EC}{EC}$$

$$\Rightarrow \quad \frac{AC}{FC} = \frac{AC}{EC}$$

$$\Rightarrow \quad \frac{1}{FC} = \frac{1}{EC}$$

$$\Rightarrow \quad FC = EC$$

This shows that *E* and *F* must coincide, but it contradicts our supposition that $E \neq F$ and $DF \parallel BC$. Hence, there is one and only line, $DE \parallel BC$, *i.e.,*

$$\boxed{l \parallel BC}$$

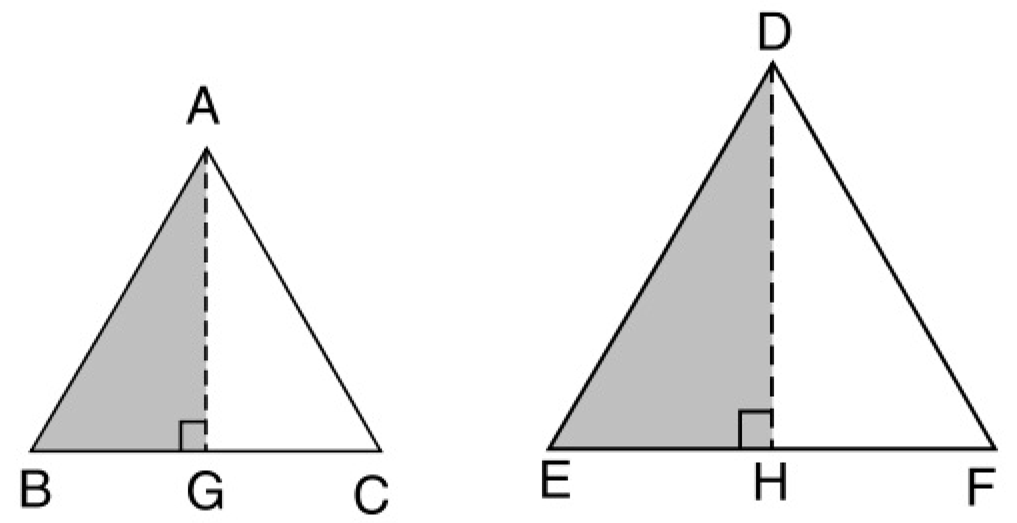
Hence Proved.

3. Ratio of the areas of two similar triangles

Statement : The ratio of the areas of the two similar triangles is equal to the ratio of squares of their corresponding sides.

Given : $\Delta ABC \sim \Delta DEF$

To Prove : $$\frac{\text{Area of }(\Delta ABC)}{\text{Area of }(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$$



Construction : Draw $AG \perp BC$ and $DH \perp EF$.

Proof :

$$\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{\frac{1}{2}\times BC\times AG}{\frac{1}{2}\times EF\times DH} \quad \text{...(i)}$$

$$= \frac{BC}{EF}\times\frac{AG}{DH} \quad \left(\text{area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height}\right)$$

Now in triangle ABG and DEH, we have

$$\angle B = \angle E \quad (\text{since, } \Delta ABC \sim \Delta DEF)$$

$$\angle AGB = \angle DHE \quad (\text{each } 90°)$$

Therefore, $\Delta ABG \sim \Delta DEH$ (by AA criterion)

Hence, $$\frac{AB}{DE} = \frac{AG}{DH} \quad \text{(Similar triangles) ...(ii)}$$

$\Rightarrow$ $$\frac{AB}{DE} = \frac{BC}{EF} \quad (\text{since, } \Delta ABC \sim \Delta DEF) \text{ ...(iii)}$$

Comparing (ii) and (iii), we get

$$\frac{AG}{DH} = \frac{BC}{EF} \quad \text{...(iv)}$$

Using (i) and (iv), we get

$$\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{BC}{EF}\times\frac{BC}{EF} = \frac{BC^2}{EF^2} \quad \text{...(v)}$$

$$\frac{BC}{EF} = \frac{AB}{DE} = \frac{AC}{DF} \quad (\text{since, } \Delta ABC \sim \Delta DEF) \text{ ...(vi)}$$

Using (v) and (vi), we get

$$\boxed{\frac{\text{Area of }(\Delta ABC)}{\text{Area of }(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}}$$

Hence Proved.

Corollary :

1. The ratio of the areas of the two similar triangles is equal to the ratio of squares of their corresponding altitudes.
2. The ratio of the areas of the two similar triangles is equal to the ratio of squares of their corresponding medians.

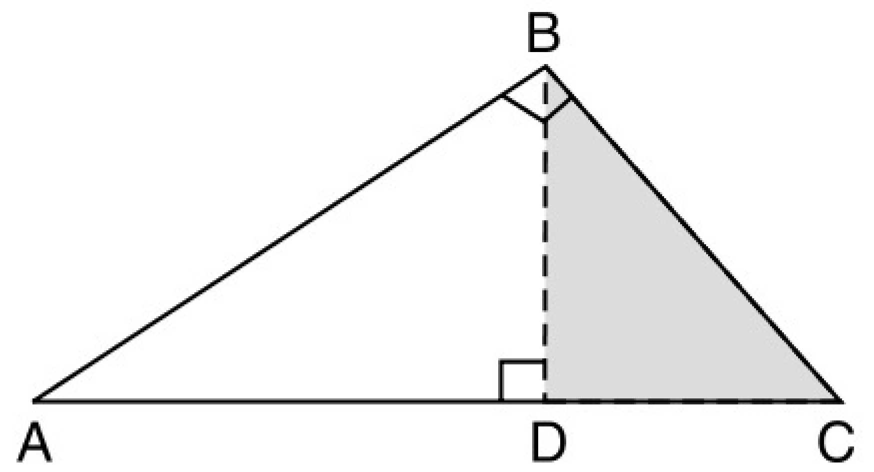
4. Pythagoras Theorem (Baudhayan Theorem)

Statement : In a right triangle, the square of one side (longest side, *i.e.*, hypotenuse) is equal to the sum of squares of other two sides (*i.e.*, base and perpendicular).

Given : ΔABC is right angled at B.

To Prove : $AC^2 = AB^2 + BC^2$.

Construction : Draw $BD \perp AC$.



Proof : Taking ΔADB and ΔABC

$\angle B = \angle ADB$ (each 90º)

$\angle A = \angle A$ (common)

Therefore, $\Delta ADB \sim \Delta ABC$ (by AA criterion)

Hence, $\dfrac{AD}{AB} = \dfrac{AB}{AC}$

$\Rightarrow$ $AB^2 = AD \times AC$ ...(i)

Now taking ΔCDB and ΔCBA

$\angle B = \angle BDC$ (each 90°)

$\angle C = \angle C$ (common)

Therefore, $\Delta CDB \sim \Delta CBA$ (by AA criterion)

Hence, $\dfrac{CD}{BC} = \dfrac{BC}{AC}$

$\Rightarrow$ $BC^2 = CD \times AC$ ...(ii)

Adding (i) and (ii), we get

$\Rightarrow$ $AB^2 + BC^2 = AD \times AC + CD \times AC$

$\Rightarrow$ $AB^2 + BC^2 = AC \times (AD + CD)$

$\Rightarrow$ $AB^2 + BC^2 = AC \times AC$

$$\boxed{AC^2 = AB^2 + BC^2}$$ **Hence Proved.**

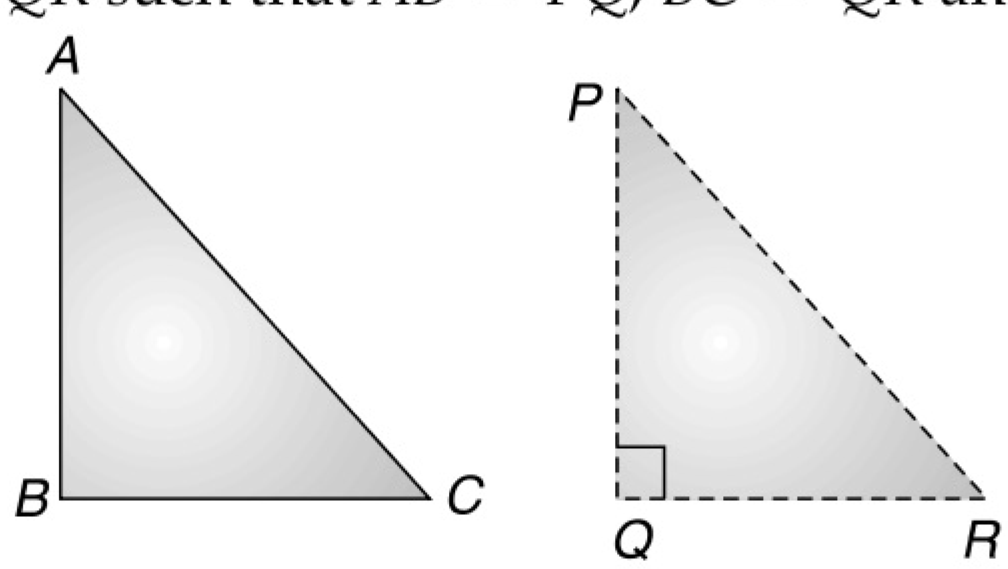
5. Converse of Pythagoras Theorem

Statement : In a triangle, if the square of one side (longest side, *i.e.* hypotenuse) is equal to the sum of squares of other two sides (*i.e.* base and perpendicular), then the angle opposite to the first side is the right angle.

Given : $AC^2 = AB^2 + BC^2$. ...(i)

To Prove : ΔABC is right angled at B.

Construction : Draw right ΔPQR such that $AB = PQ$, $BC = QR$ and $\angle Q = 90°$.



Proof : Using Pythagoras theorem in ΔPQR, we get

$$PR^2 = PQ^2 + QR^2 \quad \text{...(ii)}$$

By construction, $AB = PQ$

$BC = QR$, substituting these values in (ii), we get

$$PR^2 = AB^2 + BC^2 \quad \text{...(iii)}$$

Comparing (i) and (iii), we get

$$AC^2 = PR^2$$

$\Rightarrow$ $AC = PR$ ...(iv)

In ΔABC and ΔPQR

$AB = PQ$ (by construction)

$BC = QR$ (by construction)

$AC = PR$ (proved above in (iv))

$\Rightarrow$ $\Delta ABC = \Delta PQR$ (by SSS congruence rule)

$\Rightarrow$ $\angle B = \angle Q$ (by cpct)

But $\angle Q = 90^\circ$ (by construction)

Hence, $\angle B = 90^\circ$

ΔABC is right angled at B. **Hence Proved.**

CHAPTER 8 : Circles

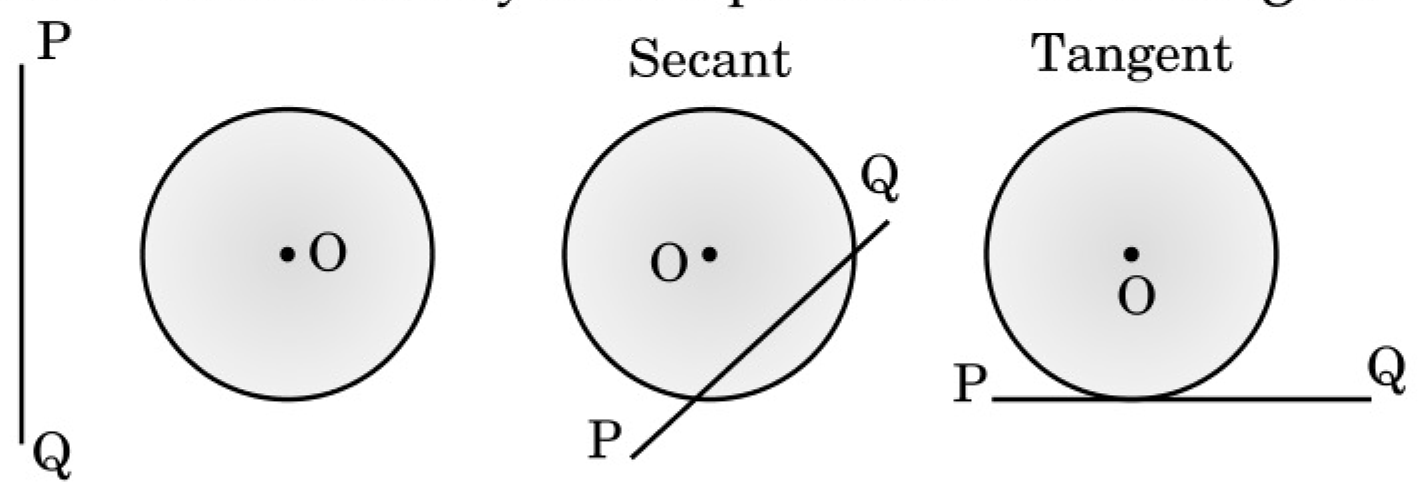
- **Fundamentals :**

Consider a circle $C(O, r)$ and a line PQ. There can be three possibilities given below :

(i) Non intersecting line w.r.t. circle

(ii) A line intersects circle in two distinct points, this line is called a **Secant**.

(iii) A line which intersects circle exactly at one point is called a **Tangent**.



From a point P inside a circle, the number of tangents drawn to the circle = 0.

From a point P on a circle, the number of tangents drawn to the circle = 1.

From a point P outside the circle, the number of tangents drawn to the circle = 2.

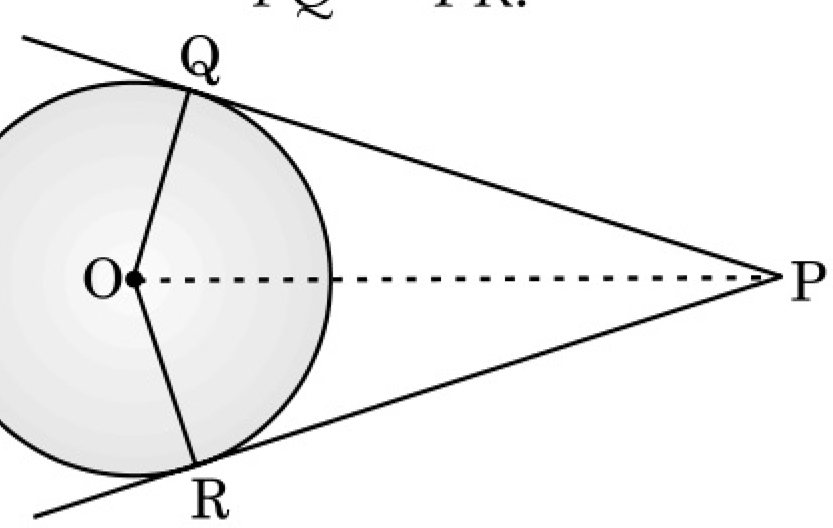
The distance between two parallel tangents drawn is equal to the diameter of the circle.

Theorem 1 : The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Theorem 2 : The lengths of two tangents from an external point to a circle are equal.

Given : A circle $C(O, r)$ and two tangents say PQ and PR from an external point P.

To prove : $PQ = PR$.



Construction : Join OQ, OR and OP.

Proof : In ΔOQP and ΔORP

$OQ = OR$ (radii of the same circle)

$OP = OP$ (Common)

$\angle Q = \angle R$ = each 90° (The tangent at any point of a circle is perpendicular to the radius through the point of contact)

Hence $\Delta OQP \cong \Delta ORP$ (By RHS Criterion)

$\therefore$ $PQ = PR$ (By c.p.c.t.)

Hence Proved.

Theorem 3 : Tangents are equally inclined on the line segment joining external point and centre.

Theorem 4 : Tangent subtend equal angle at the centre.

CHAPTER 9 : Constructions

- **To divide a line segment internally in a given ratio $m : n$, where both m and n are positive integers.**

  Steps of Construction :

  Step 1 : Draw a line segment AB of given length by using a ruler.

  Step 2 : Draw any ray AX making an acute angle with AB.

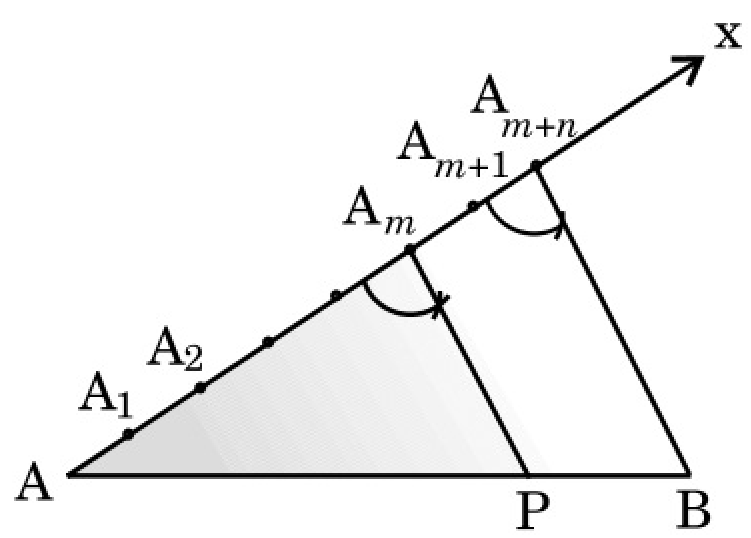
  Step 3 : Along AX mark off $(m + n)$ points $A_1, A_2, \ldots\ldots\ldots A_m, A_{m+1}. \ldots\ldots\ldots A_{m+n}$, such that $AA_1 = A_1A_2 = A_{m+n-1} A_{m+n}$.

  Step 4 : Join BA_{m+n}.

  Step 5 : Through the point A_m draw a line parallel to $A_{m+n}B$ by making an angle equal to $\angle AA_{m+n}B$ at A_m.

  Suppose this line meets AB at point P.

  The point P so obtained is the required point which divides AB internally in the ratio $m : n$.



- **Construction of triangles similar to a given triangle :**

  Steps of Construction : (a) when $m < n$,

  Step 1 : Construct the given triangle ABC by using the given data.

  Step 2 : Take any one of the three sides of the given triangle as base. Let AB be the base of the given triangle.

  Step 3 : At one end, say A, of base AB, Construct an acute $\angle BAX$ below the base AB.

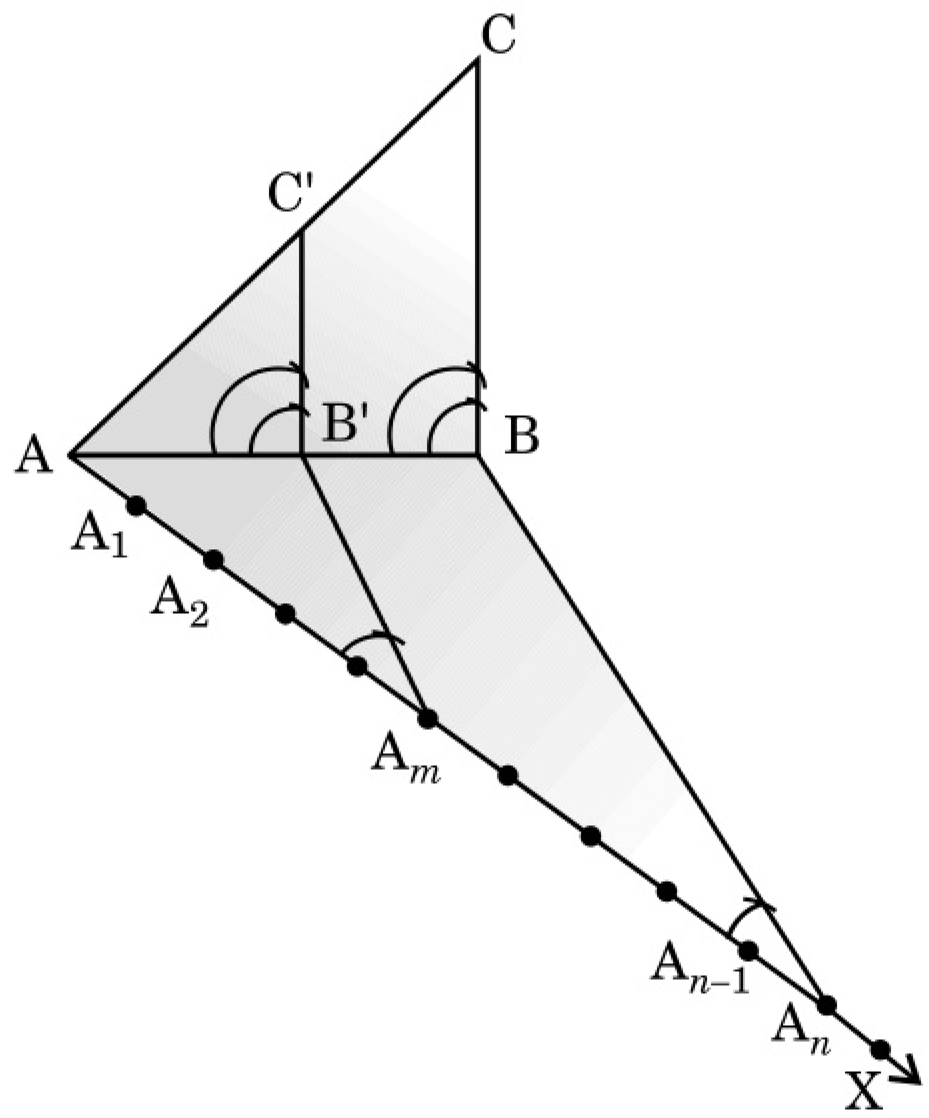
  Step 4 : Along AX mark off n points $A_1, A_2, A_3, \ldots\ldots\ldots\ldots A_n$ such that $AA_1 = A_1A_2 = \ldots\ldots\ldots\ldots = A_{n-1}A_n$.

  Step 5 : Join A_nB.

  Step 6 : Draw A_mB parallel to A_nB which meets AB at B'.

  Step 7 : From B' draw $B'C' \parallel CB$ meeting AC at C'.

Triangle $AB'C'$ is the required triangle each of whose sides is $\left(\frac{m}{n}\right)^{th}$ of the corresponding side of ΔABC.



Steps of Construction : (b) when *m* > *n*,

Step 1 : Construct the given triangle by using the given data.

Step 2 : Take any one of the three sides of the given triangle and consider it as the base. Let AB be the base of the given triangle.

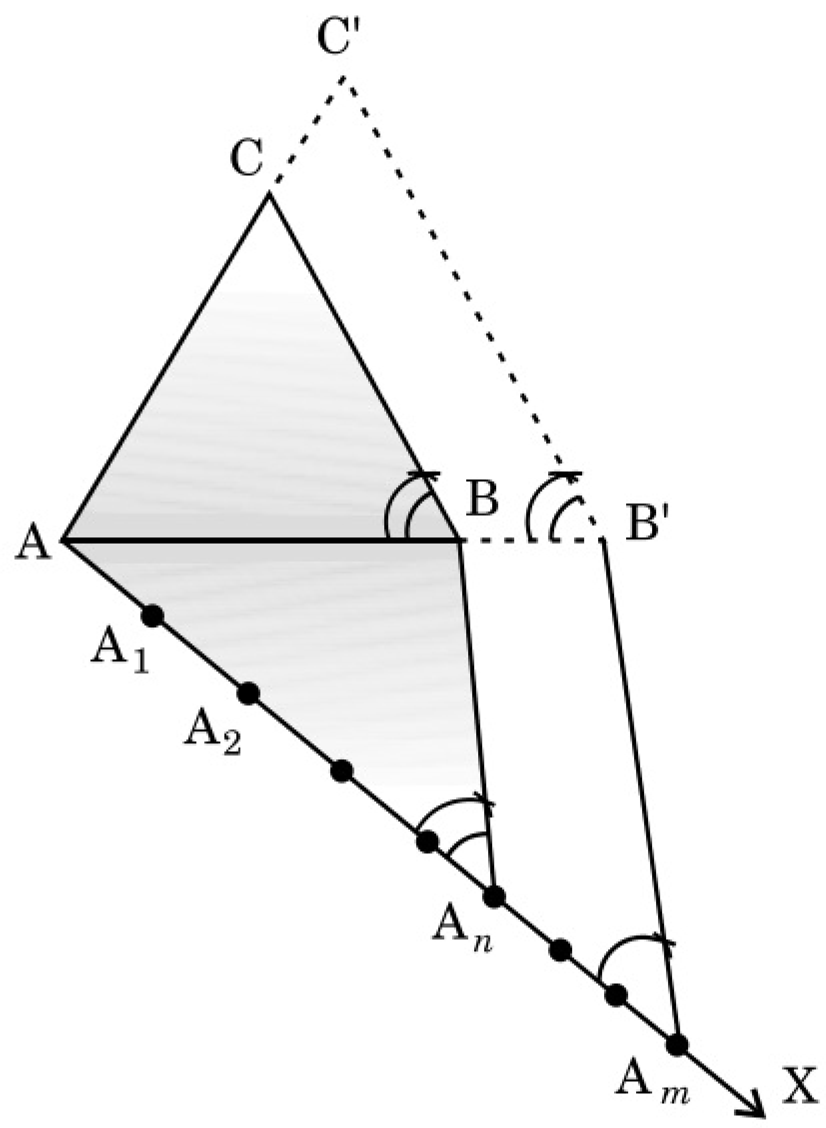
Step 3 : At one end, say A, of base AB, construct an acute $\angle BAX$ below base AB *i.e.*, on the opposite side of the vertex C.

Step 4 : Along AX mark off m (larger of m and n) points $A_1, A_2, A_3,$ ……….. A_m such that $AA_1 = A_1A_2 =$ …………$= A_{m-1}A_m$.

Step 5 : Join A_nB to B and draw a line through A_m parallel to A_nB, intersecting the extended line segment AB at B',

Step 6 : Draw a line through B' parallel to BC intersecting the extended line segment AC at C'.

Step 7 : $\Delta AB'C'$ so obtained is the required triangles, each of whose sides is $\left(\frac{m}{n}\right)^{\text{th}}$ of the corresponding side of ΔABC.

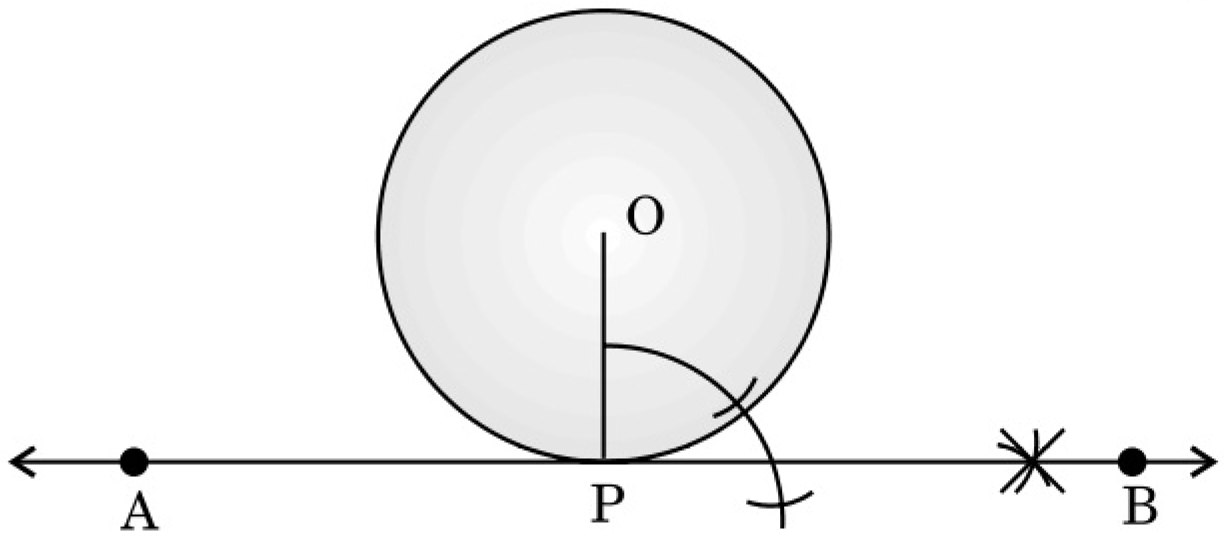


- **To draw the tangent to a circle at a given point on it, when the centre of the circle is known.**
  Given : A circle with centre *O* and a point *P* on it.
  Required : To draw the tangent to the circle at *P*.

Steps of construction :

(i) Join *OP*,

(ii) Draw a line *AB* perpendicular to *OP* at the point *P*, *APB* is the required tangent at *P*,



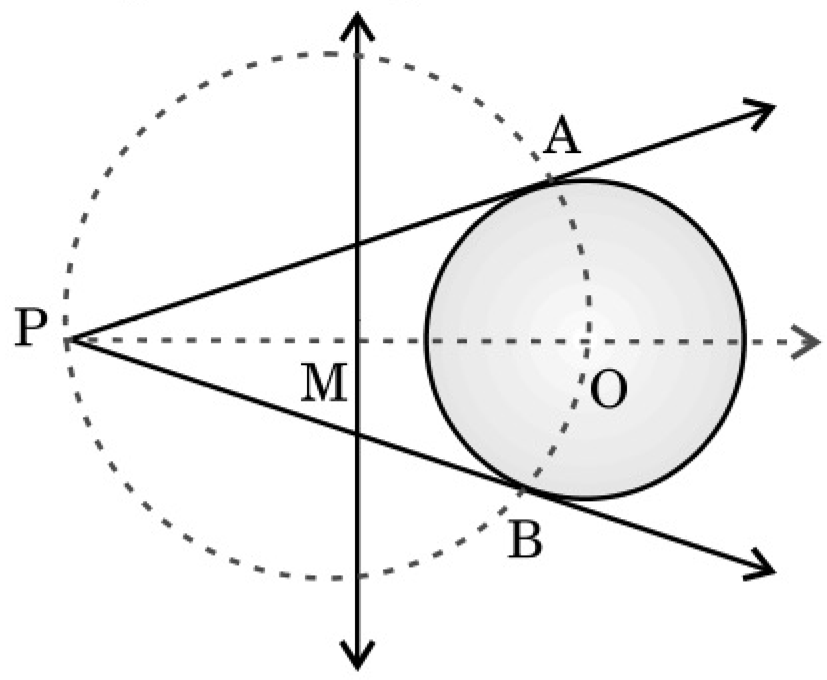
- **To draw the tangent to a circle from a point outside it (external point) when its centre is known.**
  Given : A circle with centre *O* and a point *P* outside it.
  Required : To construct the tangents to the circle from *P*.

Steps of construction :

(i) Join *OP* and bisect it. Let *M* be the mid point of *OP*.

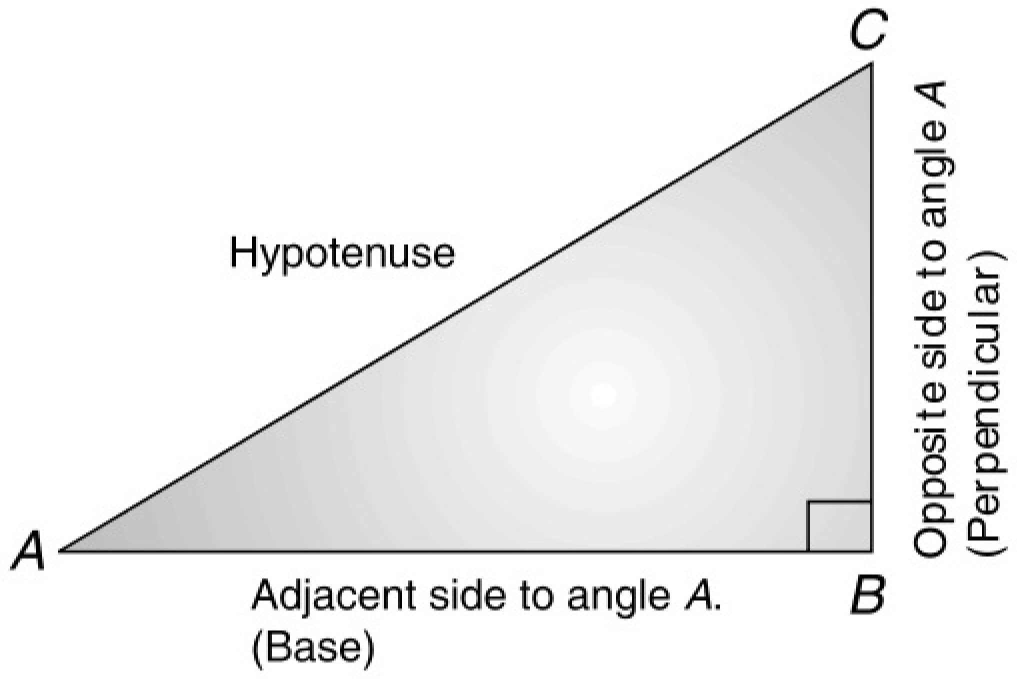
(ii) Taking *M* as centre and *MO* as radius, draw a circle to intersect *C*(*O*, *r*) in two points, say *A* and *B*.

(iii) Join *PA* and *PB*. These are the required tangents from *P* to *C*(*O*, *r*).



CHAPTER 10 : Introduction to Trigonometry and Trigonometric Identities

- Trigonometry is the branch of mathematics dealing with the relations of the sides and angles of triangles and with the relevant functions of any angles.
- **Trigonometric Ratios :** The values of the ratios of the sides of any right triangle with respect to any angle (other than 90°) are called trigonometric ratios of that angle. For example : In right $\angle ABC$, the ratios of the sides of the triangle with respect to $\angle A$ are called trigonometric ratios of $\angle A$.



There are six different trigonometric ratios as follows :

1. Sine $A = \dfrac{\text{Opposite side to angle } A}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$
2. Cosine $A = \dfrac{\text{Adjacent side to angle } A}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{\text{Base}}{\text{Hypotenuse}}$
3. Tangent $A = \dfrac{\text{Opposite side to angle } A}{\text{Adjacent side to angle } A} = \dfrac{BC}{AB} = \dfrac{\text{Perpendicular}}{\text{Base}}$
4. Cosecant $A = \dfrac{\text{Hypotenuse}}{\text{Opposite side to angle } A} = \dfrac{AC}{BC} = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$
5. Secant $A = \dfrac{\text{Hypotenuse}}{\text{Adjacent side to angle } A} = \dfrac{AC}{AB} = \dfrac{\text{Hypotenuse}}{\text{Base}}$
6. Cotangent $A = \dfrac{\text{Adjacent side to angle } A}{\text{Opposite side to angle } A} = \dfrac{AB}{BC} = \dfrac{\text{Base}}{\text{Perpendicular}}$

Tips :

1. sin A is written for sine A.
2. cos A is written for cosine A.
3. tan A is written for tangent A.
4. cosec A is written for cosecant A.
5. sec A is written for secant A.
6. cot A is written for cotangent A.

Relation between Trigonometric Ratios :

$\sin\theta = \dfrac{1}{\text{cosec}\,\theta}$ OR $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$ OR $\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{1}{\cot\theta}$ OR $\cot\theta = \dfrac{1}{\tan\theta}$

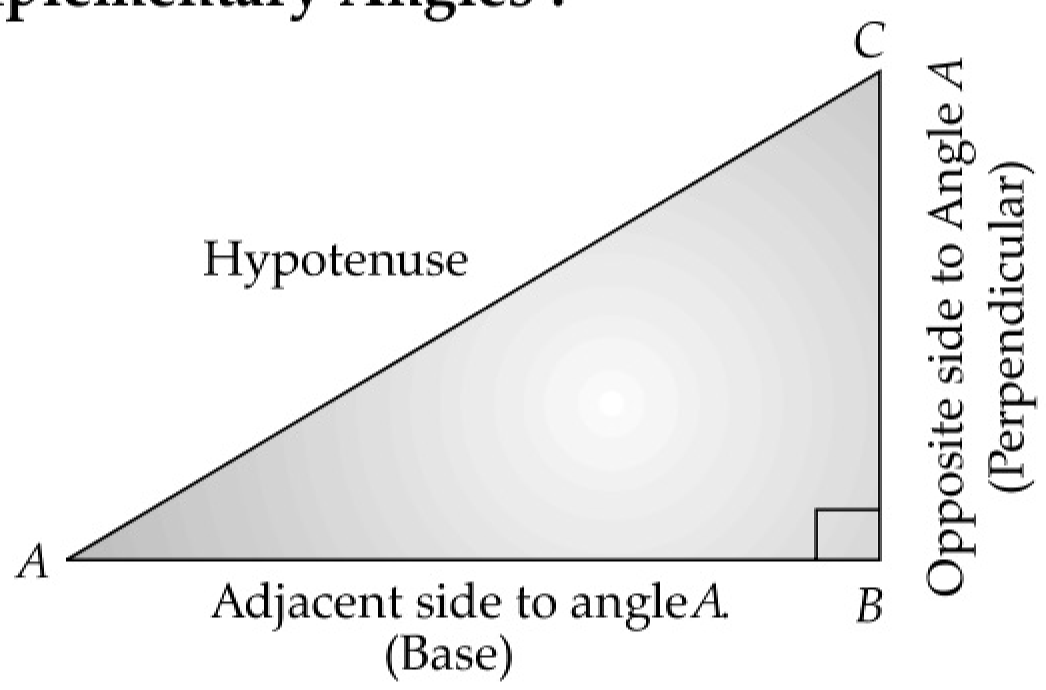
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ OR $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

- **Trigonometric Ratios of Some Specific Angles :**

In this part, we will put values of angles as 0°, 30°, 45°, 60° and 90°, hence we will find ratios.

θ	**0°**	**30°**	**45°**	**60°**	**90°**
$\sin\theta$	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
$\text{cosec}\,\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0

- **Trigonometric Ratios of Complementary Angles :**



In ΔABC, $\angle B = 90°$, Let $\angle A = \theta$, hence $\angle C = 90° - \theta$.
Thus angles θ and $(90° - \theta)$ are complementary angles.

1. $\sin(90° - \theta) = \cos\theta$ OR $\cos(90° - \theta) = \sin\theta$
2. $\text{cosec}\,(90° - \theta) = \sec\theta$ OR $\sec(90° - \theta) = \text{cosec}\,\theta$
3. $\tan(90° - \theta) = \cot\theta$ OR $\cot(90° - \theta) = \tan\theta$

- **Fundamental Trigonometric Identities :**

There are three fundamental identities which can be written in six different ways.

1. $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta = 1 - \cos^2\theta$$
$$\cos^2\theta = 1 - \sin^2\theta$$

2. $\sec^2\theta = 1 + \tan^2\theta$

$$\tan^2\theta = \sec^2\theta - 1$$
$$\sec^2\theta - \tan^2\theta = 1$$

3. $\text{cosec}^2\theta = 1 + \cot^2\theta$

$$\cot^2\theta = \text{cosec}^2\theta - 1$$
$$\text{cosec}^2\theta - \cot^2\theta = 1$$

Proof of first identity : $\sin^2\theta + \cos^2\theta = 1$

Let P denotes Perpendicular, B denotes Base, and H denotes Hypotenuse.

We know that, $\sin\theta = \frac{P}{H}$ and $\cos\theta = \frac{B}{H}$

LHS $= \sin^2\theta + \cos^2\theta$ (putting values of sin θ and cos θ)

$$= \frac{P^2}{H^2} + \frac{B^2}{H^2}$$

⇒ $$= \frac{P^2 + B^2}{H^2}$$

$$= \frac{H^2}{H^2}$$

Therefore, $= 1 =$ R.H.S. **Hence Proved.**

Similarly, other two identities can be proved.

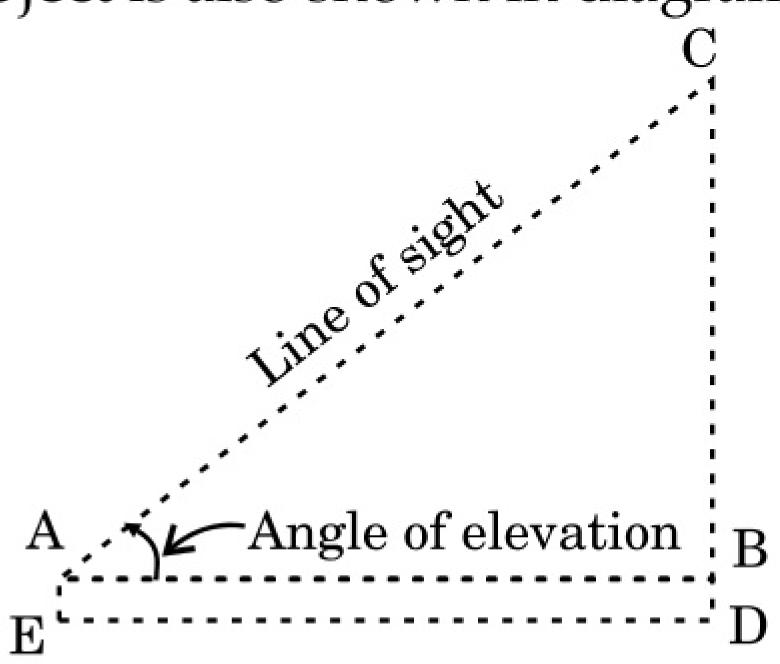
CHAPTER 11 : Heights and Distances

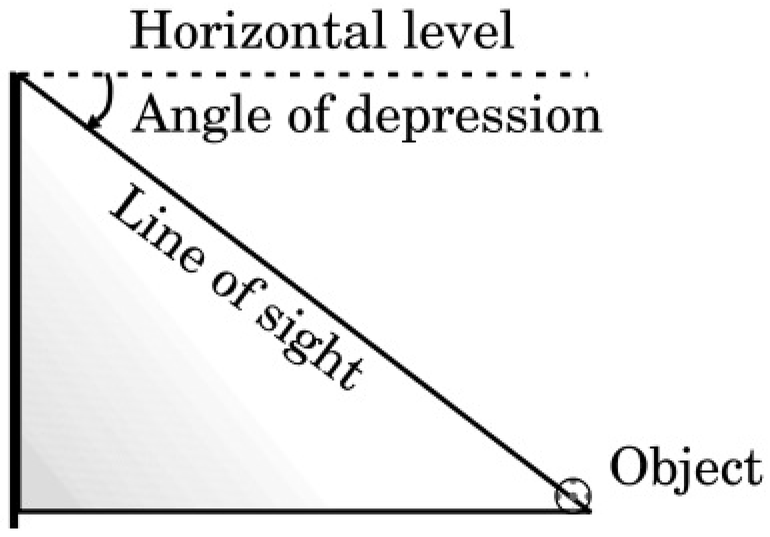
- **Angle of Elevation :**

The **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.

The **angle of elevation** of the point viewed is the angle formed by the line of sight with the horizontal.

When the point being viewed is above the horizontal level, *i.e.*, the case when we raise our head to look at the object is also shown in diagram that we have to assume a horizontal level at our eyes.





- **Angle of Depression :**

The **angle of depression** of a point on the object being viewed is the angle formed by the line of sight with the horizontal.

When the point is below the horizontal level, *i.e.*,the case when we lower our head to look at the point being viewed.

CHAPTER 12 : Areas Related to Circles

- **Fundamentals :**

1. Circle is defined as the set of all those points which are at a constant distance from a fixed point. The fixed point is called **centre**.
2. The constant distance is called **radius**.
3. The longest chord passing through centre and whose end point lies on circle is called **diameter**.
4. Circles with same centre are called **Concentric Circles**.
5. Perimeter of circle is called circumference.
6. π is defined as the ratio of circumference and diameter of circle.

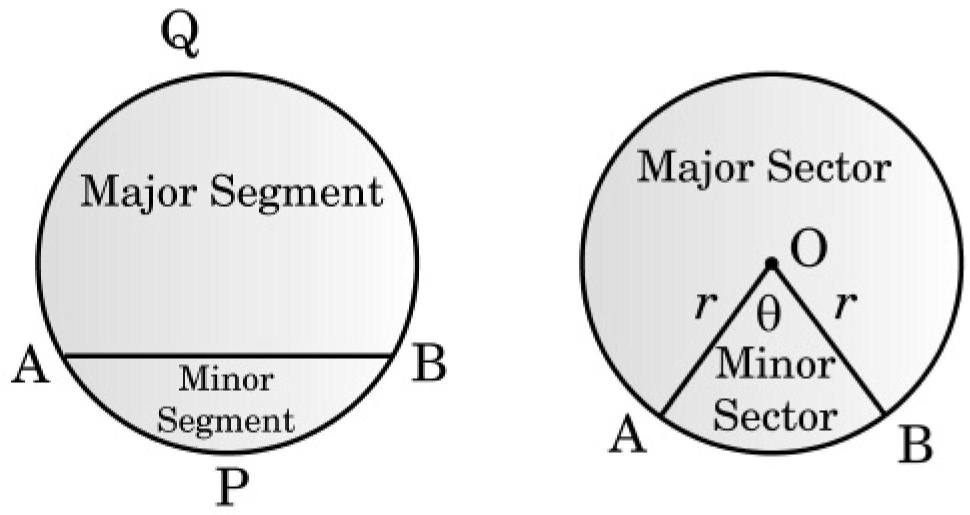
i.e., $$\pi = \frac{\text{Circumference}}{\text{Diameter}}$$

$\therefore$ Circumference $= \pi \times$ Diameter

$\Rightarrow$ Circumference $= 2\pi r$ [where, r is the radius of circle]

and Area of the Circle $= \pi r^2$

7. **Arc, Chord, Segment, Sector of a Circle**



(i) **Arc :** Any portion of circumference. *i.e.,* APB is minor arc while AQB is major arc.

(ii) **Chord :** The line joining any two points on the circle. *i.e.,* AB.

(iii) **Segment :** In figure, chord AB divides the circle in two segments *i.e.,* $APBA$ (minor segment) and $AQBA$ (major segment).

(iv) **Sector :** The region bounded by the two radii AO and BO and arc AB is called sector of the circle.

8. **Length of Arc :** When sector angle $\angle AOB = \theta$. (where, θ is called central angle.)

We know that length of arc when sector angle ($\angle AOB = 360°$) is $2\pi r$

length of arc when sector angle ($\angle AOB = 1°$) is $= \frac{2\pi r}{360°}$

Length of arc when sector angle ($\angle AOB = \theta$) $= \frac{2\pi r \times \theta}{360°}$

$$\text{Length of arc } AB = 2\pi r \times \frac{\theta}{360°}$$

$$\text{Length of sector} = \frac{1}{2} \times l \times r$$

l = length of arc

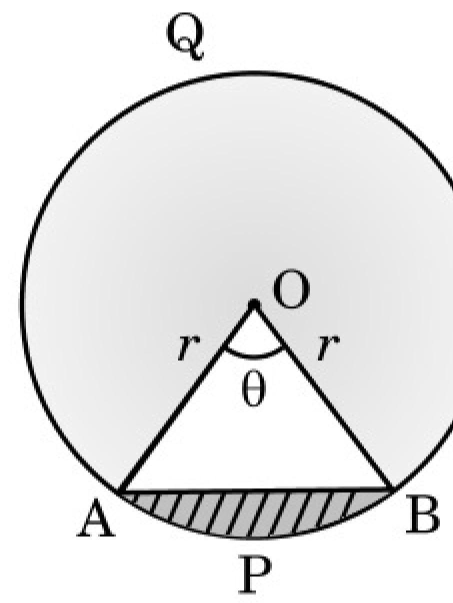
r = radius

9. **Area of Sector :** When sector angle $\angle AOB = \theta$

We know that area of circle when sector angle ($\angle AOB = 360°$) is πr^2

Area of arc when sector angle ($\angle AOB = 1°$) is $\dfrac{\pi r^2}{360°}$

Area of arc when sector angle ($\angle AOB = \theta$) is $\dfrac{\pi r^2}{360°} \times \theta$



$\therefore$ $$\text{Area of Sector} = \pi r^2 \times \frac{\theta}{360°}$$

$$\text{Area of Sector} = \frac{1}{2} \times l \times r$$

l = length of arc

r = radius

10. **Area of Segment (shaded) of A Circle :** Area of Sector AOB – Area of triangle AOB

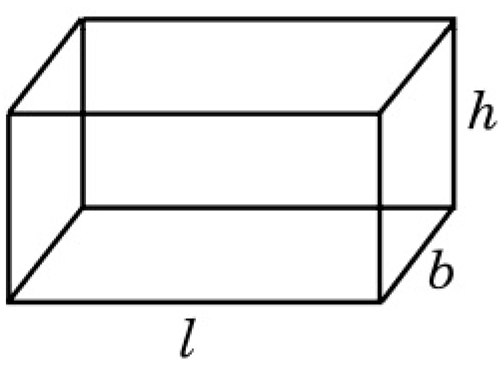
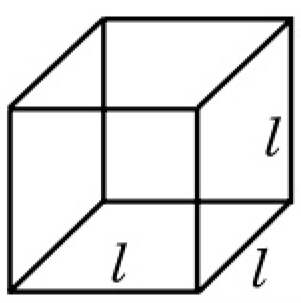
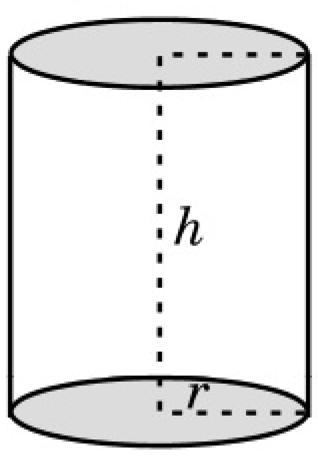
We can use formula given below when $\theta \geq 90°$	We can use formula given below when $\theta \leq 90°$
Area of segment $= \dfrac{\pi r^2 \theta}{360°} - r^2 \sin \dfrac{\theta}{2} \cos \dfrac{\theta}{360°}$	Area of segment $= \dfrac{\pi r^2 \theta}{360°} - \dfrac{1}{2} r^2 \sin \theta$

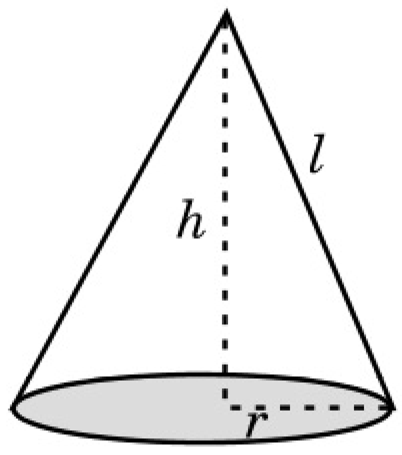
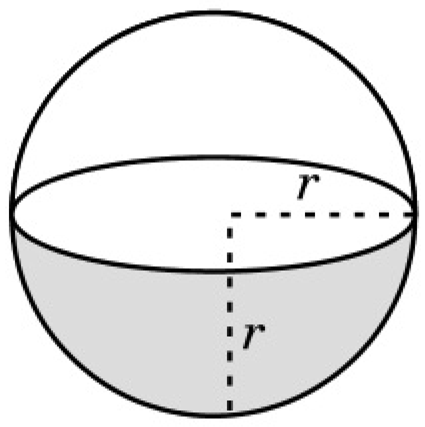
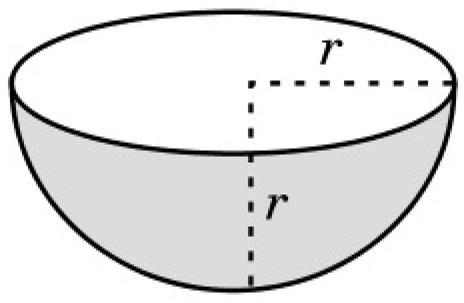
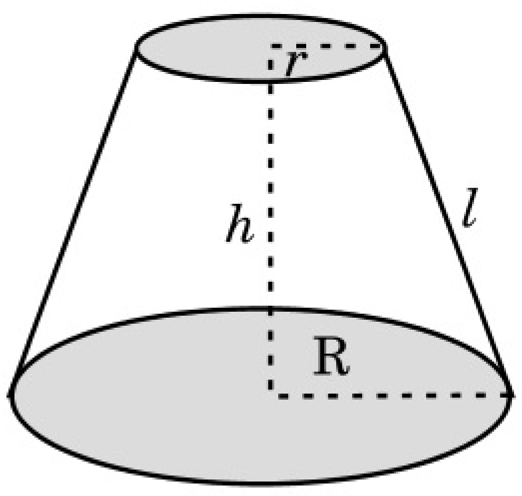
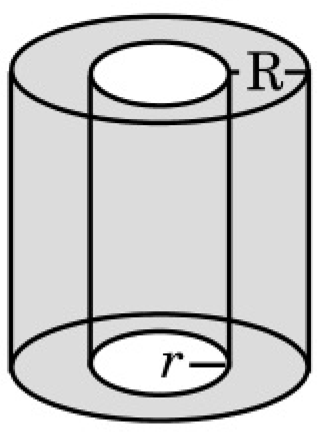
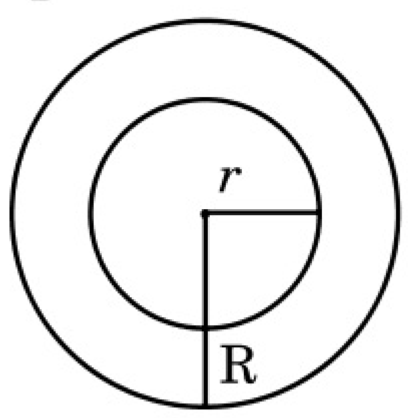
11. **Perimeter Of Segment (shaded) Of A Circle :** AB + arc(APB)

Perimeter of Segment $= \dfrac{2\pi r\theta}{360°} + 2r \sin \dfrac{\pi\theta}{360°}$

CHAPTER 13 : Surface Areas and Volumes

Fundamentals :

S. No.	Shape	CSA	TSA	Volume	Nomenclature
1.	**Cuboid** 	$2(bh + hl)$	$2(lb+bh+hl)$	lbh	l = length b = breadth h = height
2.	**Cube** 	$4l^2$	$6l^2$	l^3	l = length or side
3.	**Right Circular Cylinder** 	$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$	r = radius of base h = height

4.	**Right Circular Cone**  	πrl	$\pi r(r + l)$	$\frac{1}{3}(\pi r^2 h)$	r = radius of base h = height l = slant height $l = \sqrt{r^2 + h^2}$
5.	**Sphere**  	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}(\pi r^3)$	r = radius
6.	**Hemisphere**  	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}(\pi r^3)$	r = radius
7.	**Frustum**  	$\pi l(r + R)$	$\pi l(r + R)$ + $\pi(r^2 + R^2)$	$\frac{1}{3}\pi h(r^2 + R^2 + rR)$	r = radius of smaller base R = radius of larger base h = height l = slant height $l = \sqrt{h^2 + (R - r)^2}$
8.	**Right Circular Hollow Cylinder**  	$2\pi h(r + R)$	$2\pi(r + R)$ $(h + R - r)$	$\pi h(R^2 - r^2)$	r = inner radius R = outer radius h = height
9.	**Spherical Shell**  	$4\pi r^2$ [Internal] $4\pi R^2$ [External]	$4\pi r^2 + 4\pi R^2$	$\frac{4}{3}[\pi(R^3 - r^3)]$	r = inner radius R = outer radius

$$\text{Area} \times \text{Rate} = \text{Cost}$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$1\text{ m}^3 = 1000\text{ L}$$

$$1\text{ m}^3 = 1\text{ kL}$$

$$1\text{ L} = 1000\text{ cm}^3$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$1\text{ km} = 1000\text{ m} = 10^5\text{ cm}$$

$$1\text{ km}^2 = 10^6\text{ m}^2$$

$$1\text{ m} = 100\text{ cm}$$

$$1\text{ m}^2 = 10000\text{ cm}^2$$

$$1\text{ km/hr} = \frac{5}{18}\text{ m/sec}$$

$$1\text{ km/hr} = \frac{50}{3}\text{ m/min}$$

Shape of river = Cuboid

$$1\text{ acre} = 100\text{ m}^2$$

$$1\text{ hectare} = 10000\text{ m}^2$$

CHAPTER 14 : Statistics

- **Fundamentals :**

1. The word **statistics** is used in both singular as well as plural.
2. In singular, it means "science of collection, presentation, analysis and interpretation of numerical data".
3. In plural, it means "numerical facts collected with definite purpose".
4. The number of times an observation occurs in the given data is called the frequency.
5. Frequency distribution is of two types :
   (i) Discrete Frequency distribution
   (ii) Continuous or Grouped Frequency distribution
6. Class Mark = (Lower limit + Upper Limit)/2.
7. The commonly used measures of central tendency are as follows :
   Arithmetic Mean (MEAN), Geometric Mean, Harmonic Mean, Median and Mode.

(i) Relation between mean, median and mode :

$$3\text{ Median} = \text{Mode} + 2\text{ Mean}$$

(ii) Mean of Grouped Data : If $x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n$, are observations with respective frequencies $f_1, f_2, f_3, \ldots\ldots\ldots\ldots\ldots f_n$, it means observation x_1 occurs f_1 times, observation x_2 occurs f_2 times and so on. Mean is denoted by $\bar{x}$.

There are three different ways to find the mean of a grouped data which are :

(a) Direct Method.
(b) Assumed Mean Method.
(c) Shortcut Method (Step-Deviation Method).

Direct Method :

$$\text{Mean } \bar{x} = \frac{\text{Sum of all the observations}}{\text{No. of observations}}$$

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \ldots\ldots + f_nx_n}{f_1 + f_2 + \ldots\ldots + f_n}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i}$$

Assumed Mean Method : (Shortcut Method)

$$\text{Mean } (\bar{x}) = a + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$$

where, a is any arbitrary value, chosen as assumed mean (somewhere in the middle of x_i), and $d_i = x_i - a$

Step-Deviation Method :

$$\text{Mean, } (\bar{x}) = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$$

where, a is any arbitrary value, chosen as assumed mean (somewhere in the middle of x_i,

h = class-size.

and $u_i = \frac{x_i - a}{h}$

Combined Mean : If $\overline{x_1}$ and $\overline{x_2}$ are the means of two groups having same unit of measurement computed from n_1 and n_2 values.

$$\text{Mean } (\bar{x}) = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

(iii) Median of Grouped Data :

Condition I : When the data is discrete.

Step 1 : Arrange data in ascending order.

Step 2 : If the total frequency n is odd :

Then, $\left(\frac{n+1}{2}\right)^{th}$, observation is the median.

Step 3 : If the total frequency n is even :

Then, mean of $\frac{n}{2}^{\text{th}}$ and $\left(\frac{n}{2}+1\right)^{\text{th}}$, observations the median.

Condition II : When the data is continuous and in the form of frequency distribution :

Then,

$$\text{Median} = l + \left[\frac{\frac{n}{2} - c}{f}\right] \times h$$

Median class = The class whose cumulative frequency is greater than (nearest to) $\frac{N}{2}$.

where,

l = lower limit of median class

f = frequency of median class

h = class-size

n = number of observations

c = cumulative frequency of class preceding the median class.

(iv) Mode of Grouped Data : The class with maximum frequency is called the modal class.

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

where,

l = lower limit of the modal class
h = class-size
f_1 = frequency of the modal class
f_0 = frequency of the class preceding the modal class
f_2 = frequency of the class succeeding the modal class

(v) Ogive or Cumulative Frequency Curve : The term ogive is derived from the word *ogee*. An *ogee* is a shape consisting of concave arc flowing into a convex arc.

An Ogive of less than type : It is drawn for less than type cumulative frequency distribution. Here we mark upper limit of class interval on horizontal axis while respective cumulative frequency is marked on vertical axis and plot the corresponding points and join them by a free hand curve. Cumulative frequency is counted up to down.

An Ogive of more than type : It is drawn for more than type cumulative frequency distribution. Here we mark lower limit of class interval on horizontal axis while respective cumulative frequency is marked on vertical axis and plot the corresponding points and join them by a free hand curve. Cumulative frequency is counted down to up.

Note : Intersecting point of less than ogive and more than ogive gives median.

Note : The median of the grouped data can be obtained on any one of the ogive by locating $\frac{N}{2}$ on the y-axis. Locate corresponding point on the ogive, x-coordinate of that point determines the median of the data.

CHAPTER 15 : Probability

- **Fundamentals :**
  1. **Experiment :** An operation which can produce some well defined outcomes.
  2. **Sample Space :** It is the total number of possible outcomes of a random experiment.
  3. **Event :** Any subset of sample space is called event.
  4. **Elementary Event :** Each outcome of any random experiment.
  5. **Sure Event (Certain event) :** An event which always occurs whenever the random experiment is performed.
  6. **Impossible Event :** An event which never occurs whenever the random experiment is performed.
  7. **Favourable Event :** The cases which ensure the occurrence of an event.
  8. **Probability :** Probability $P(E)$ of an event E is defined as :

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total Number of outcomes}}$$

$$P(E) = \frac{\text{Favourable Event}}{\text{Sample Space}}$$

  9. **Complement Events :** An event associated with a random experiment denoted by $P(\text{not-}E)$ which happens only when E does not happen is called the complement of event E.

$$P(\overline{E}) \text{ or } P(\text{not } E) = 1 - P(E)$$

- **Tips :**
  1. Sum of the probabilities of all the elementary events of an experiment is 1.

$$P(E_1) + P(E_2) + P(E_3) + \ldots\ldots\ldots\ldots\ldots + P(E_n) = 1,$$

  2. Probability of Sure Event is 1.
  3. Probability of an Impossible Event is 0.
  4. Probability of any event lies between 0 and 1 (including 0 and 1) *i.e.*,

$$0 \leq P\,(E) \leq 1.$$

  5. 52 cards are divided into 4 suits of 13 cards is each. The suits are :

SPADE ♠ **HEARTS** ♥

DIAMONDS ♦ **CLUBS** ♣

  6. Out of 52 cards 26 are red in colour and 26 are black.
  7. In each suit there is an Ace, a King, a Queen, a Jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2.
  8. King, Queen and Jack are called face cards.