

## The Principle of Mathematical Induction

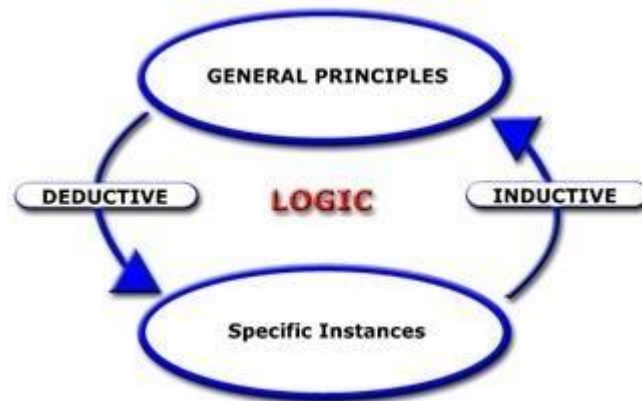
**Deduction:** Generalization of Specific Instance

Example : Rohit is a man & All men eat food, therefore, Rohit eats food.

**Induction:** Specific Instances to Generalization

Example : Rohit eats food. Vikash eats food. Rohit and Vikash are men. Then, All men eat food

Statement is true for  $n=1$ ,  $n=k$  &  $n=k+1$ , then, the Statement is true for all natural numbers  $n$ .



### Steps of Principle of Mathematical Induction:

**Step 1:** Let  $P(n)$  be a result or statement formulated in terms of  $n$ (given question).

**Step 2:** Prove that  $P(1)$  is true

**Step 3:** Assume that  $P(k)$  is true

**Step 4:** Using Step 3, prove that  $P(k+1)$  is true

**Step 5:** Thus  $P(1)$  is true and  $P(k+1)$  is true whenever  $P(k)$  is true.

Hence, by the Principle of Mathematical Induction,  $P(n)$  is true for all natural numbers  $n$ .

**Example:** Prove that  $2^n > n$  for all positive integers  $n$

**Solution:**

Step 1: Let  $P(n)$ :  $2^n > n$

Step 2: When  $n = 1$ ,  $2^1 > 1$ . Hence  $P(1)$  is true.

Step 3: Assume that  $P(k)$  is true for any positive integer  $k$ , i.e.,  $2^k > k$  ... (1)

Step 4: We shall now prove that  $P(k+1)$  is true whenever  $P(k)$  is true.

Multiplying both sides of (1) by 2, we get

$$2 * 2^k > 2 * k$$

$$\text{i.e., } 2^{k+1} > 2k$$

$$\text{or, } 2^{k+1} > k + k$$

$$\text{or, } 2^{k+1} > k + 1 \quad (\text{since } k > 1)$$

Therefore,  $P(k+1)$  is true when  $P(k)$  is true. Hence, by principle of mathematical induction,  $P(n)$  is true for every positive integer  $n$ .

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