12
Chapter

Areas Related to Circles

Key Points

1. **Circle:** A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point always remains the same. The fixed point is called the centre and the given constant distance is known as the radius of the circle.

   If r is radius of a circle, then

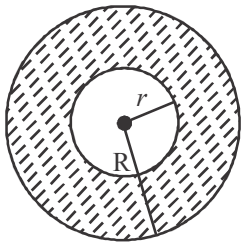
   (i) Circumference = $2\pi r$ or πd where d = 2r is the diameter of the circle

   (ii) Area = πr^2 or $\frac{\pi d^2}{4}$

   (iii) Area of semi circle = $\frac{\pi r^2}{2}$

   (iv) Area of quadrant of a circle = $\frac{\pi r^2}{4}$

   Area enclosed by two concentric circles: If R and r are radii of two concentric circles, then area enclosed by the two circles = $\pi R^2 - \pi r^2$



$= \pi (R^2 - r^2)$

$= \pi (R + r)(R - r)$

   (i) If two circles touch internally, then the distance between their centres is equal to the difference of their radii.

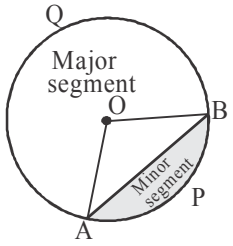
   (ii) If two circles touch externally, then distance between their centres is equal to the sum of their radii.

(iii) Distance moved by rotating wheel in one revolution is equal to the circumference of the wheel.

(iv) The number of revolutions completed by a rotating wheel in

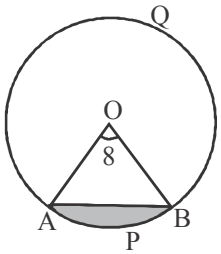
$$\text{one minute} = \frac{\textit{Distance moved in one minute}}{\textit{Circumference of the wheel}}$$

Segment of a Circle: The portion (or part) of a circular region enclosed between a chord and the corresponding arc is called a segment of the circle. In fig. adjacent APB is minor segment and AQB is major segment.



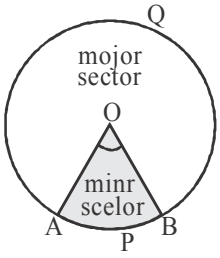
Area of segment APB = Area of the sector OAPB – Area of ΔOAB

$$= \frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$



Sector of a circle: The portion (or part) of the circular region enclosed by the two radii and the corresponding arc is called a sector of the circle.

In figure adjacent OAPB is minor sector and OAQB is the major sector.



Area of the sector of angle $\theta = \frac{\theta}{360°} \times 2\pi r^2$

$= \frac{1}{2} \times \textit{length of arc} \times \textit{radius} = \frac{1}{2} lr$

Length of an arc of a sector of angle $\theta = \frac{\theta}{360} \times 2\pi r$

(i) The sum of the arcs of major and minor sectors of a circle is equal to the circumference of the circle.

(ii) The sum of the areas of major and minor sectors of a circle is equal to the area of the circle.

(a) Angle described by minute hand in 60 minutes = 360°

Angle described by minute hand in one minute $= \frac{360°}{60°} = 6°$

Thus minute hand rotates through an angle of 6° in one minute

(b) Angle described by hour hand in 12 hours = 360°

Angle described by hour hand in one hour $= \frac{360°}{12°} = 30°$

Angle described by hour hand in one minute $= \frac{30°}{60°} = \frac{1°}{2}$

Thus, hour hand rotates through an angle of $\frac{1°}{2}$ in one minute.

VERY SHORT ANSWER QUESTIONS

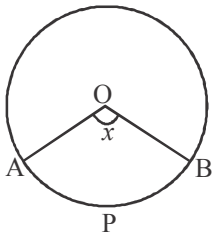
1. If the diameter of a semi circular protactor is 14 cm, then find its perimeter.
2. If circumference and the area of a circle are numerically equal, find the diameter of the circle.
3. Find the area of the circle 'inscribed' in a square of side *a* cm.
4. Find the area of a sector of a circle whose radius is *r* and length of the arc is *l*.
5. The radius of a wheel is 0.25 m. Find the number of revolutions it will make to travel a distance of 11 kms.

6. If the area of circle is 616 cm², then what is its circumference?
7. What is the area of the circle that can be inscribe in a square of side 6 cm?
8. What is the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 24 cm and 7 cm?
9. A wire can be bent in the form of a circle of radius 35 cm. If it is bent in the form of a square, then what will be its area?
10. What is the angle subtended at the centre of a circle of radius 6 cm by an arc of length 3π cm?
11. Write the formula for the area of sector of angle θ (in degrees) of a circle of radius r.
12. If the circumference of two circles are in the ratio 2:3, what is the ratio of their areas?
13. If the difference between the circumference and radius of a circle is 37 cm, then find the circumference of the circle. (Use $\pi = \frac{22}{7}$)
14. If diameter of a circle is increased by 40%, find by how much percentage its area increases?
15. The hour hand of a clock is 6 cm long. Find the area swept by it between 11:20 am and 11:55 am.

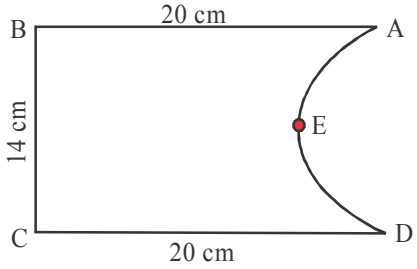
SHORT ANSWER TYPE I QUESTIONS

16. Find the area of a quadrant of a circle whose circumference is 22 cm.
17. What is the angle subtended at the centre of a circle of radius 10 cm by an arc of length 5π cm?
18. If a square is inscribed in a circle, what is the ratio of the area of the circle and the square?
19. Find the radius of semicircle if its perimeter is 18 cm.
20. If the perimeter of a circle is equal to that of square, then find the ratio of their areas.
21. What is the ratio of the areas of a circle and an equilateral triangle whose diameter and a side are respectively equal?

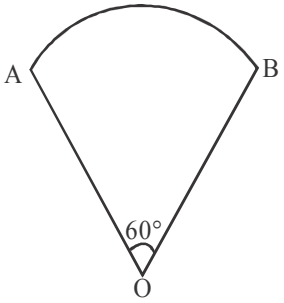
22. In fig., O is the centre of a circle. The area of sector OAPB is $\frac{5}{18}$ of the area of the circle. Find x.



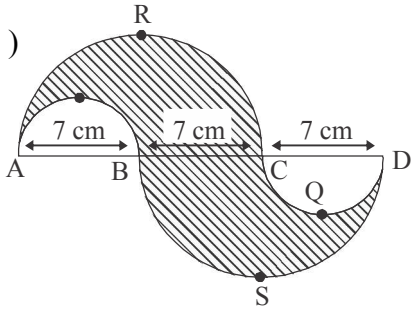
23. Find the perimeter of a given fig, where AED is a semicircle and ABCD is a rectangle.



24. In fig, is a sector of a circle of radius 10.5 cm. Find the perimeter of the sector.



25. In the given fig, APB and CQD are semi circles of diameter 7 cm each, while ARC and BSD are semicircles of diameter 14 cm each. Find the perimeter of the shaded region. (Use $\pi = \frac{22}{7}$)

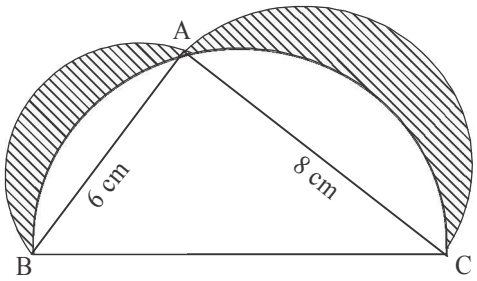


SHORT ANSWER TYPE II QUESTIONS

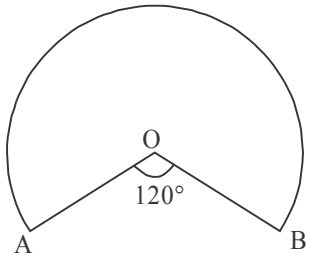
26. Area of a sector of a circle of radius 36 cm is 54π cm^2 . Find the length of the corresponding arc of the sector.

27. The length of the minute hand of a clock is 5 cm. Find the area swept by the minute hand during the time period 6:05 am to 6:40 am.

28. In fig, ABC is a triangle right angled at A. Semi circles are drawn on AB, AC and BC as diameters. Find the area of the shaded region.



29. In fig, OAPB is a sector of a circle of radius 3.5 cm with the centre at O and $\angle AOB = 120°$. Find the length of OAPBO.



30. Circular footpath of width 2 m is constructed at the rate of Rs 20 per square meter, around a circular park of radius 1500 m. Find the total cost of construction of the foot path. (Take $\pi = 3.14$)

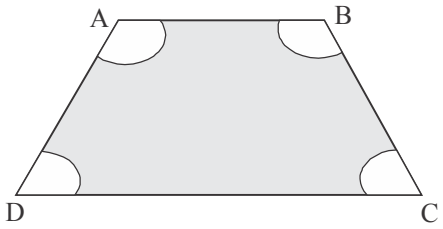
31. A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60 cm. Calculate the speed of cycle.

32. In a circle with centre O and radius 5 cm, AB is a chord of length $5\sqrt{3}$ cm. Find the area of sector AOB.

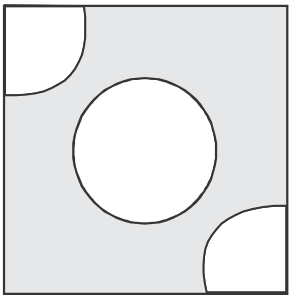
33. The area of an equilateral triangle is $49\sqrt{3}$ cm². Taking each angular point as centre, a circle is described with radius equal to half the length of the side of the triangle. Find the area of the triangle not included in the circle.

34. ABCD is a trapezium with AB||DC, AB= 18 cm, DC= 32 cm and the distance between AB and DC is 14 cm. Circles of equal radii 7 cm with centres A, B, C and D have been drawn, Then, find the area of the shaded region of the figure.

($\pi = \frac{22}{7}$)

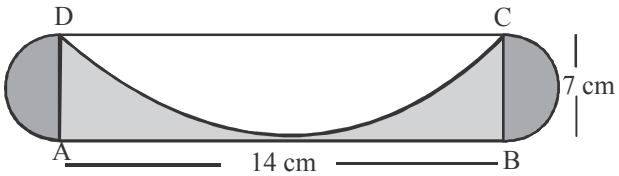


35. From each of the two opposite corners of a square of side 8 cm, a quadrant of a circle of radius 1.4 cm is cut. Another circle of radius 4.2 cm is also cut from the centre as shown in fig. Find the area of the shaded portion. (Use $\pi = \frac{22}{7}$)



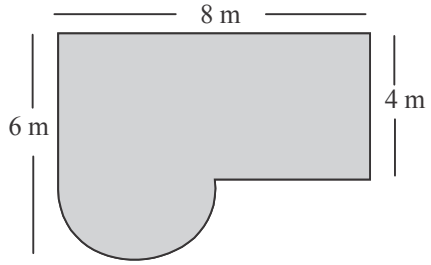
36. A sector of 100° cut off from a circle contains 70.65 cm². Find the radius of the circle. ($\pi = 3.14$)

37. In fig. ABCD is a rectangle with AB= 14 cm and BC= 7 cm. Taking DC, BC and AD as diameter, three semicircles are drawn. Find the area of the shaded portion.



38. A square water tank has its each side equal to 40 m. There are four semi circular grassy plots all around it. Find the cost of turfing the plot at Rs 1.25 per sq. m. (Use $\pi = 3.14$)

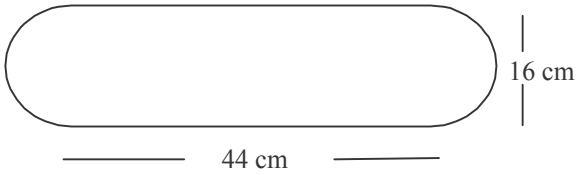
39. Find the area of the shaded region shown in the fig.



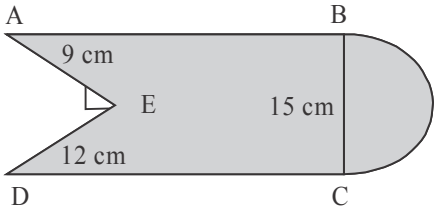
40. Find the area of the minor segment of a circle of radius28 cm, when the angle of the corresponding sector is 45°.

41. A piece of wire 11 cm long is bent into the form of an arc of a circle subtending an angle of 45° at its centre. Find the radius of the circle.

42. Find the area of the flower bed (with semicircular ends).

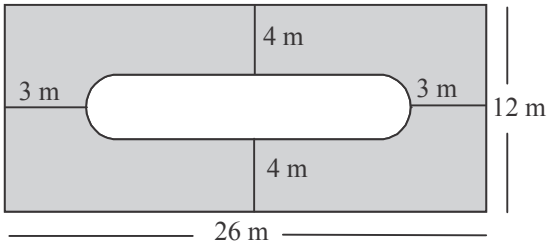


43. In fig. from a rectangular region ABCD with AB= 20 cm, a right triangle AED with AE= 9 cm and DE= 12 cm, is cut off. On the other end, taking BC as diameter, a semi circle is added on outside the region. Find the area of the shaded region.



44. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.

45. Find the area of the shaded region.



LONG ANSWER TYPE QUESTIONS

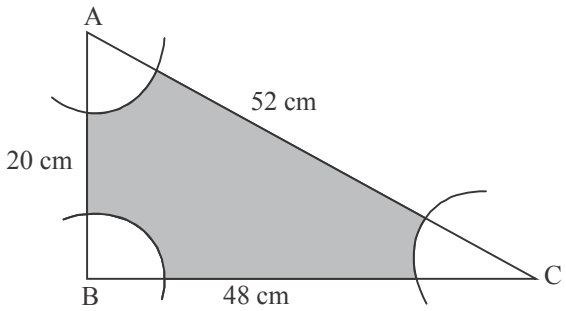
46. Two circles touch externally. The sum of their areas is 130π sq. cm and the distance between their centres is 14 cm. Find the radii of the circles.

47. Three circles each of radius 7 cm are drawn in such a way that each of their touches the other two. Find the area enclosed between the circles.

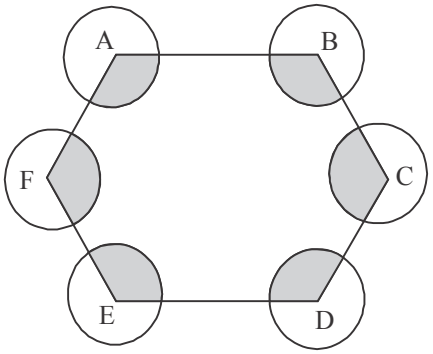
48. Find the number of revolutions made by a circular wheel of area 6.16 m^2 in rolling a distance of 572 m.

49. All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if area of the circle is 2464 cm^2.

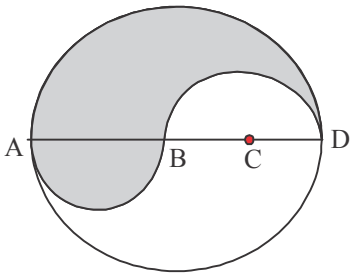
50. With vertices A, B and C of a triangle ABC as centres, arcs are drawn with radii 6 cm each in fig. If AB= 20 cm, BC= 48 cm and CA= 52 cm, then find the area of the shaded region. (Use $\pi = 3.14$)



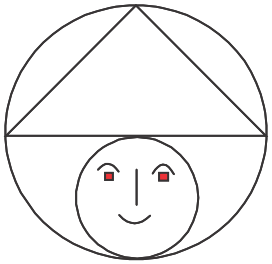
51. ABCDEF is a regular hexagon. With vertices A, B, C, D, E and F as the centres, circles of same radius '*r*' are drawn. Find the area of the shaded portion shown in the given figure.



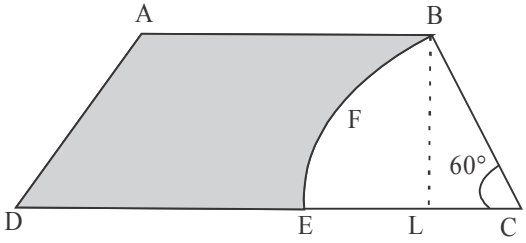
52. ABCD is a diameter of a circle of radius 6 cm. The lengths AB, BC and CD are equal. Semicircles are drawn on AB and BD as diameter as shown in the fig. Find the perimeter and area of the shaded region.



53. A poor artist on the street makes funny cartoons for children and earns his living. Once he made a comic face by drawing a circle within a circle, the radius of the bigger circle being 30 cm and that of smaller being 20 cm as shown in the figure. What is the area of the cap givn in this figure? What qualities of this artist are being reflected here?



54. In the given fig., ABCD is a trapezium with AB||CD and , $\angle BCD = 60°$, If BFEC is a sector of a circle with centre C and AB = BC = 7 cm and DE = 4 cm, then find the area of the shaded region. (Use $\pi = \frac{22}{7}$, $\sqrt{3} = 1.732$)



55. Find the area of the shaded region in the given figure.