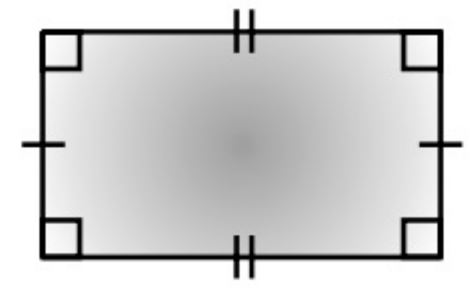
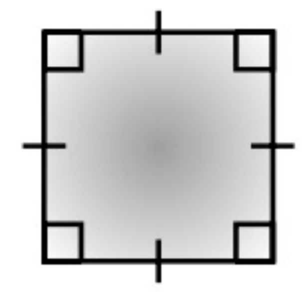
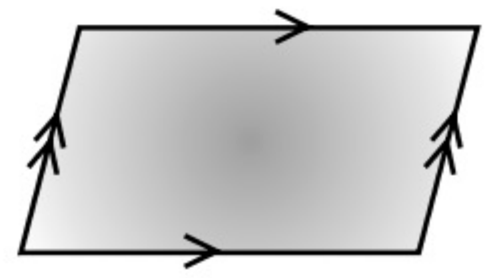
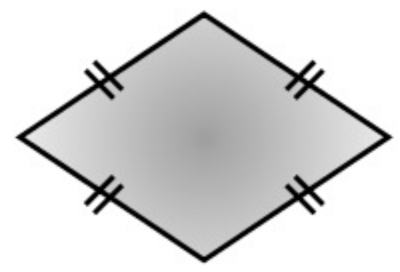
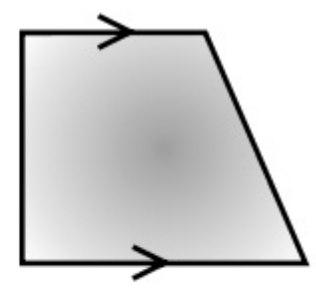
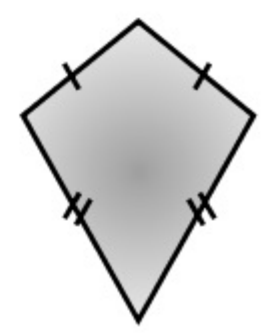
CHAPTER 8 : Quadrilaterals

Quadrilateral	Properties
Rectangle	4 right angles and pair of opposite sides equal 
Square	4 right angles and 4 equal sides 
Parallelogram	Two pairs of parallel sides and opposite sides equal 
Rhombus	Parallelogram with 4 equal sides 
Trapezium	Two sides are parallel 
Kite	Two pairs of adjacent sides of the same length 

Theorems and Proofs (wherever required) :

Theorem : 1

Statement : A diagonal of a parallelogram divides it into two congruent triangles.

Given : Parallelogram *ABCD*.

To prove : $\Delta BAC \cong \Delta DCA$.

Construction : Draw diagonal *AC*.

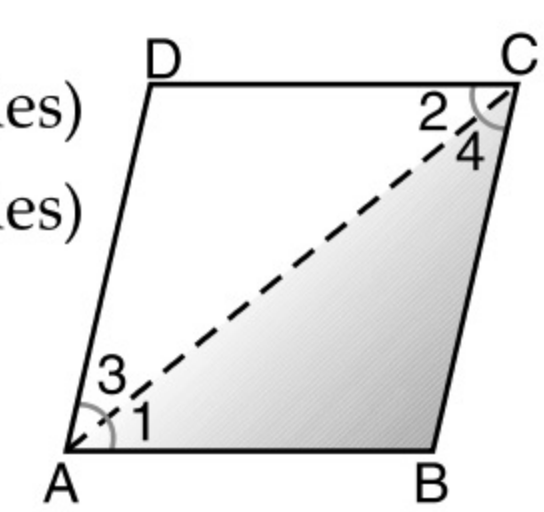
Proof : In ΔBAC and ΔDCA,

$\angle 1 = \angle 2$ (alternate interior angles)

$\angle 3 = \angle 4$ (alternate interior angles)

$AC = AC$ (common)

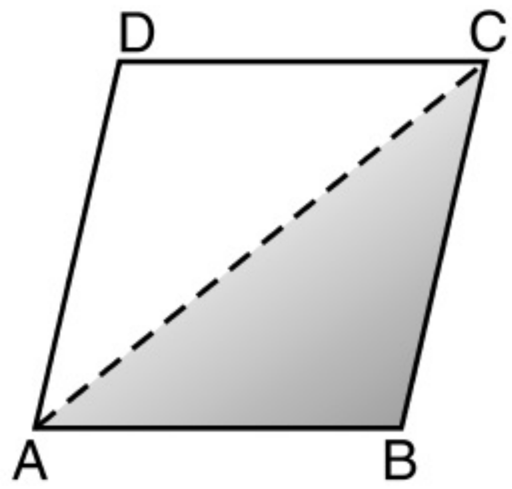
$\Delta BAC \cong \Delta DCA$, (ASA)



Theorem : 2

Statement : In a parallelogram, opposite sides are equal.

Given : Parallelogram $ABCD$.



To prove :

$$AB = DC$$
$$AD = BC$$

Construction : Draw diagonal AC.

Proof : In ΔABC and ΔCDA,

$AB \parallel DC$ (since opposite sides of a parallelogram are parallel)

$\angle BAC = \angle DCA$ (since these are alternate interior angles made by the transversal AC upon two parallel lines AB and DC)

Similarly, $BC \parallel AD$ (since opposite sides of a parallelogram are parallel)

$\angle BCA = \angle DAC$ (since these are alternate interior angles made by the transversal AC upon two parallel lines BC and AD)

Also, AC of $\Delta ABC = AC$ of ΔCDA (common sides)

$\therefore$ $\Delta ABC \cong \Delta CDA$ (ASA)

Thus, $AB = DC$ (c.p.c.t.)

and $AD = BC$ (c.p.c.t.)

Theorem : 3

Statement : If each pair of opposite sides of a quadrilateral are equal, then it is a parallelogram.

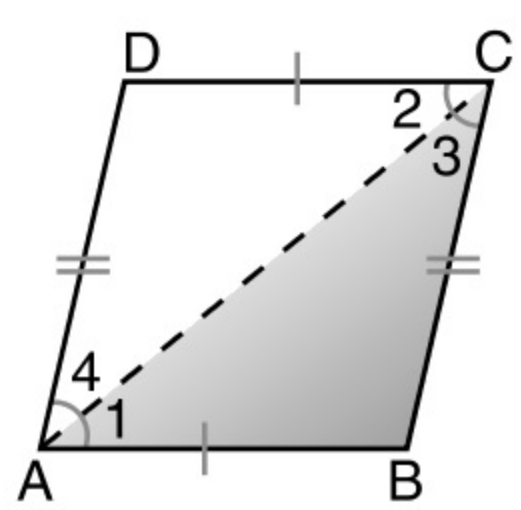
Given : A quadrilateral $ABCD$.

$$AB = CD \text{ and } BC = DA$$

To prove : Quadrilateral $ABCD$ is a parallelogram

Construction : Draw diagonal AC.

Proof : In ΔABC and ΔCDA,



$AB = CD$ (given)

$BC = DA$ (given)

$AC = CA$ (common)

$\therefore$ $\Delta ABC \cong \Delta CDA$ (SSS)

Hence, $\angle 1 = \angle 2$ (c.p.c.t.)

$\Rightarrow$ $AB \parallel CD$ ....(i)

(since $\angle 1$ and $\angle 2$ form a pair of alternate interior angles)

Also, $\angle 3 = \angle 4$ (c.p.c.t.)

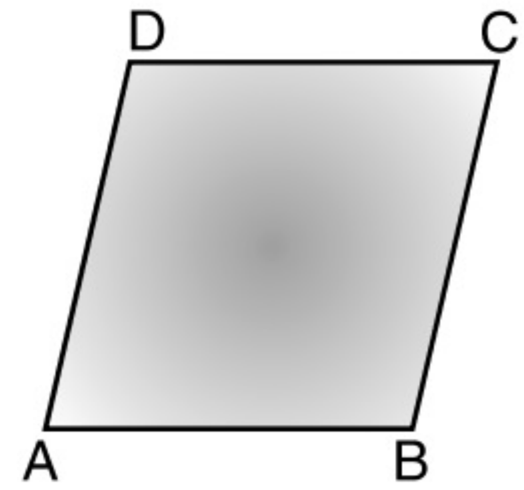
$\Rightarrow$ $BC \parallel DA$ ....(ii)

(since $\angle 3$ and $\angle 4$ form a pair of alternate interior angles)

From (i) and (ii), we get that $ABCD$ is a parallelogram by definition.

Theorem : 4

Statement : In a parallelogram, opposite angles are equal.



Given : Parallelogram $ABCD$.

To prove :

$$\angle A = \angle C$$
$$\angle B = \angle D$$

Proof : In Parallelogram $ABCD$,

Consider $AD \parallel BC$ and AB transversal

$$\angle A + \angle B = 180^\circ \quad \text{(co-interior angles)} \quad ....(i)$$

Now, consider $AB \parallel DC$ and BC transversal

$$\angle B + \angle C = 180^\circ \quad \text{(co-interior angles)} \quad ....(ii)$$

From (i) and (ii) we get,

$$\angle A + \angle B = \angle B + \angle C$$
$$\angle A = \angle C$$

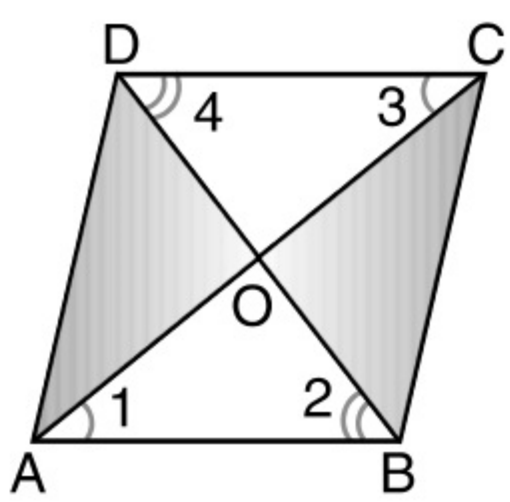
Similarly $\angle B = \angle D$

Theorem : 5

Statement : In a quadrilateral, if each pair of opposite angles are equal, then it is a parallelogram.

Theorem : 6

Statement : The diagonals of a parallelogram bisect each other.



Given : Parallelogram $ABCD$.

Diagonals AC and BD intersect at O.

To prove : AC and BD bisect each other at O.

Proof : In ΔAOB and ΔCOD,

$\angle 1 = \angle 3$ (alternate interior angles)

$AB = CD$ (opposite sides of a parallelogram)

$\angle 2 = \angle 4$ (alternate interior angles)

$\Delta AOB \cong \Delta COD$ (ASA)

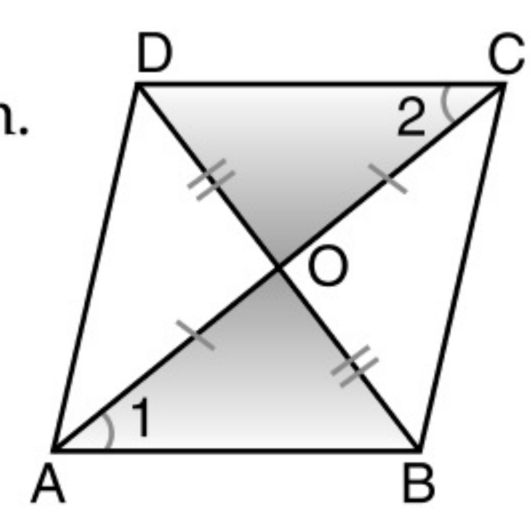
$\therefore$ $AO = CO$ (c.p.c.t.)

and $BO = DO$ (c.p.c.t.)

Thus, AC and BD bisect each other at O.

Theorem : 7

Statement : If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.



Given : Quadrilateral $ABCD$.

Diagonals AC and BD bisect each other at O.

To prove : Quadrilateral $ABCD$ is a parallelogram.

Proof : In ΔAOB and ΔCOD.

$AO = CO$ (since the diagonals bisect each other at O)

$BO = DO$ (since the diagonals bisect each other at O)

$\angle AOB = \angle COD$ (since these are vertically opposite angles)

Hence, $\Delta AOB \cong \Delta COD$ (SAS)

$\therefore$ $\angle 1 = \angle 2$ (c.p.c.t.)

But $\angle 1$ and $\angle 2$ form a pair of alternate interior angles made by the transversal AC upon two line segments AB and CD.

Hence, $AB \parallel CD$

Similarly, $BC \parallel DA$

$\therefore$ Quadrilateral $ABCD$ is a parallelogram by definition.

Theorem : 8

Statement : A quadrilateral is a parallelogram, if a pair of opposite sides are equal and parallel.

Theorem : 9

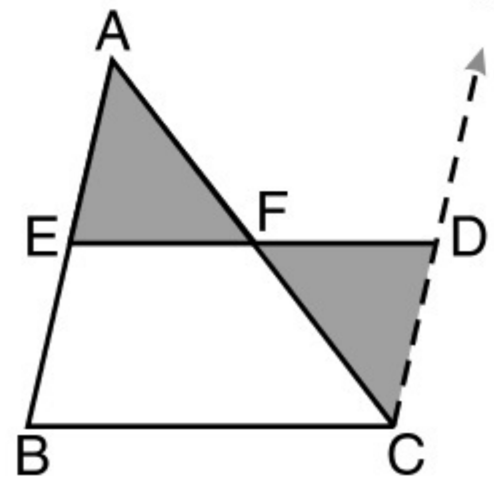
Statement : The line segment joining the mid-points of two sides of a triangle is parallel to the third side and equal to half of it.

Given : ΔABC, E and F are the mid-points of the sides AB and AC respectively.

To prove : $EF \parallel BC$ and $EF = \frac{1}{2} BC$.

Construction : Draw a line CD parallel to BA. Let it intersects EF produced at D.

Proof : In ΔAEF and ΔCDF,



$\angle EAF = \angle FCD$ (alternate interior angles)

$AF = FC$ (F is the mid-point of AC)

$\angle AFE = \angle CFD$ (vertically opposite angles)

$\therefore$ $\Delta AEF \cong \Delta CDF$ (ASA)

$EF = DF$ and $AE = DC$ (c.p.c.t.)

$\therefore$ $BE = AE = DC$ (E is the mid-point of AB)

Now $BE = DC$ and $BE \parallel DC$ (opposite sides of a parallelogram)

Therefore, $BCDE$ is a parallelogram.

This gives $EF \parallel BC$.

Also, $EF = DF$ and $EF + DF = ED = BC$

$\therefore$ $2EF = BC$

$\therefore$ $EF = \frac{1}{2} BC$

Theorem : 10

Statement : The line drawn through the mid-points of one side of a triangle, parallel to another side bisects the third side. ■■