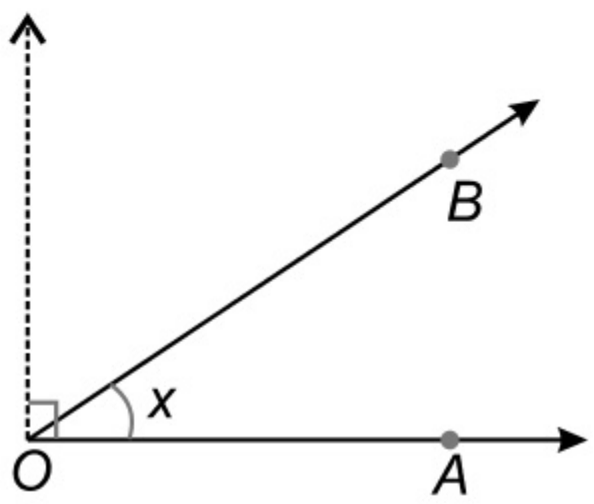
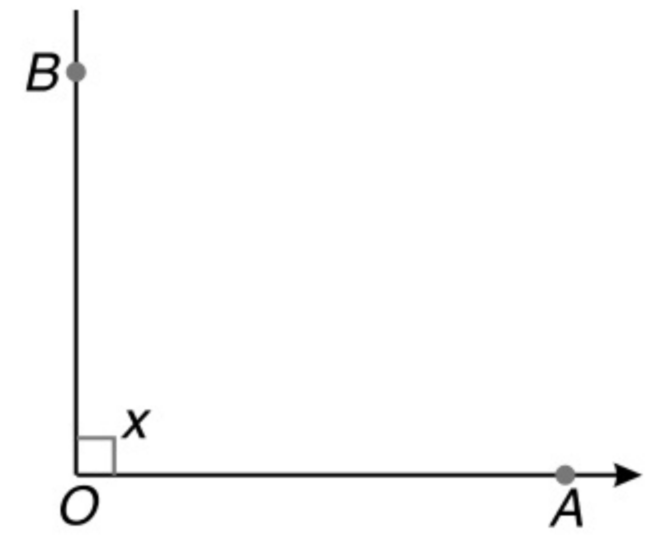
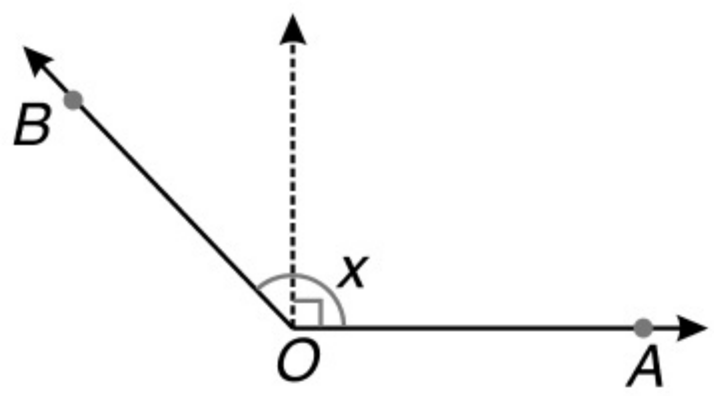
CHAPTER 6 : Lines and Angles



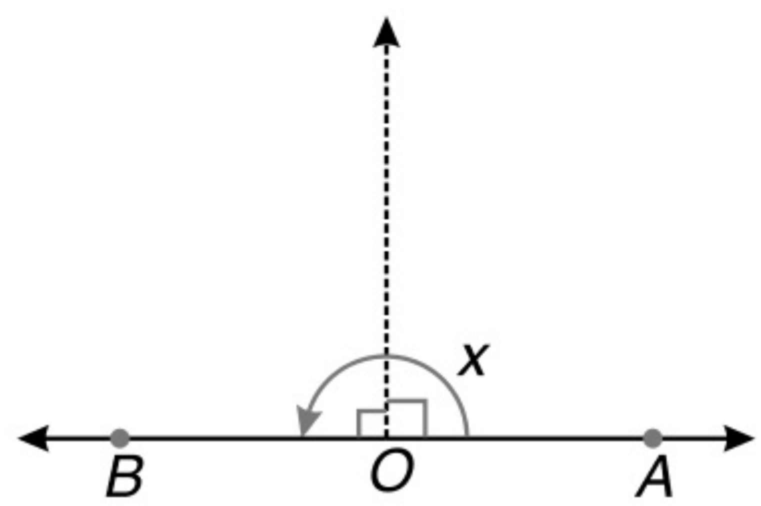
(i) **Acute angle :** $0° < x < 90°$



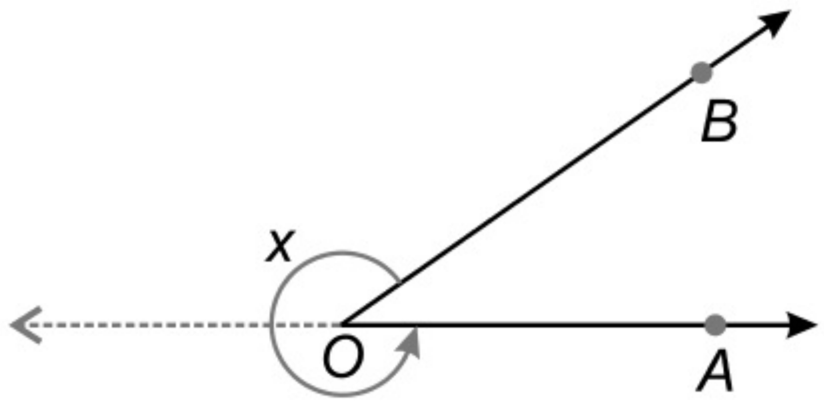
(ii) **Right angle :** $x = 90°$



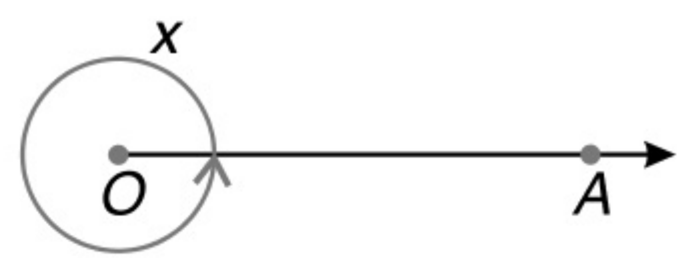
(iii) **Obtuse angle :** $90° < x < 180°$



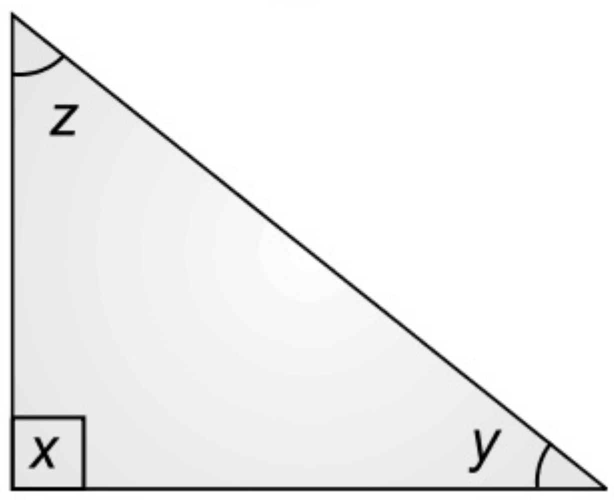
(iv) **Straight angle :** $x = 180°$



(v) **Reflex angle :** $180° < x < 360°$

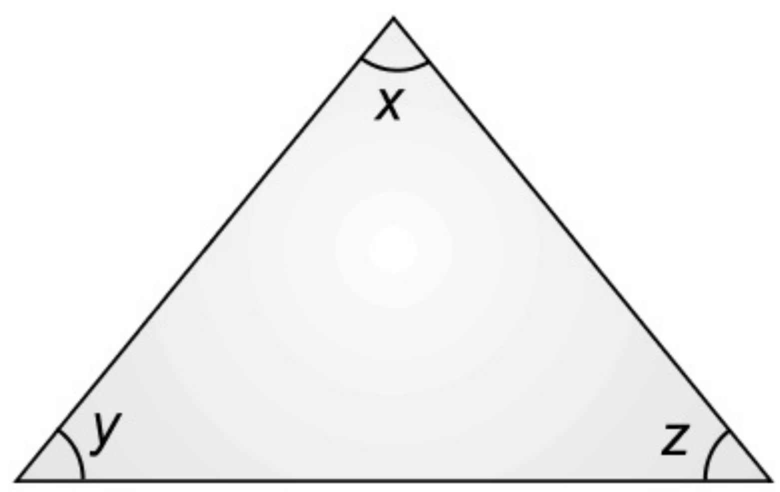


(vi) **Complete angle :** $x = 360°$



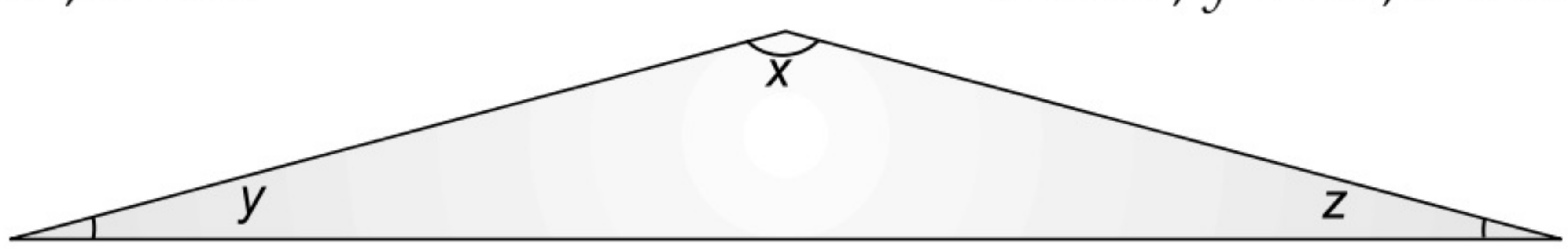
(vii) **Right Angled Triangle**

$x = 90°; y < 90°; z < 90°$



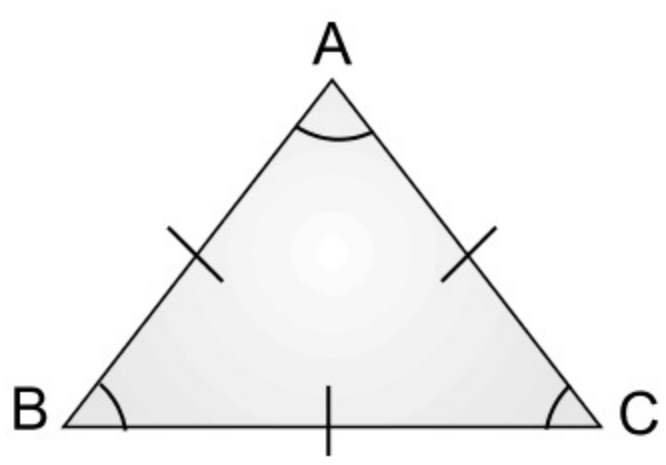
(viii) **Acute Angled Triangle**

$x < 90°; y < 90°; z < 90°$



(ix) **Obtuse Angled Triangle**

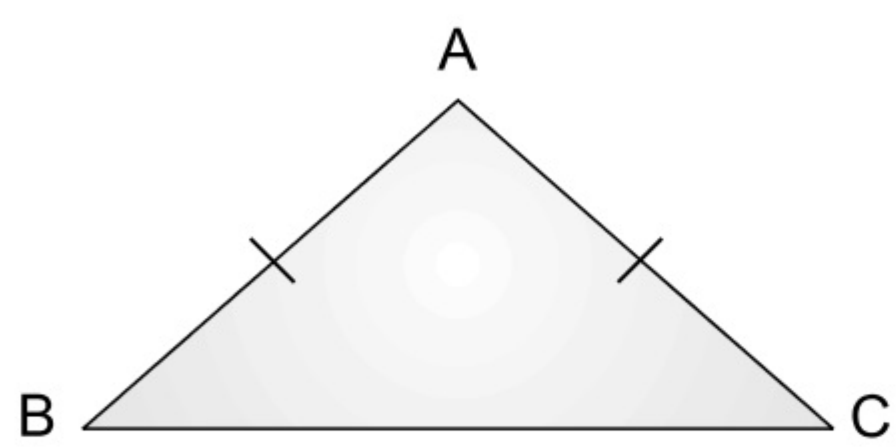
$x > 90°; y < 90°; z < 90°$



(x) **Equilateral Triangle**

$AB = BC = CA$

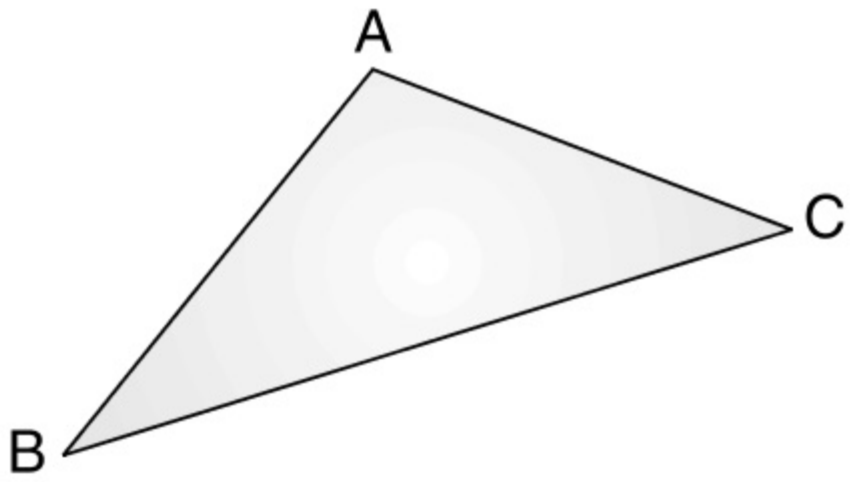
$\angle A = \angle B = \angle C = 60°$



(xi) **Isosceles Triangle**

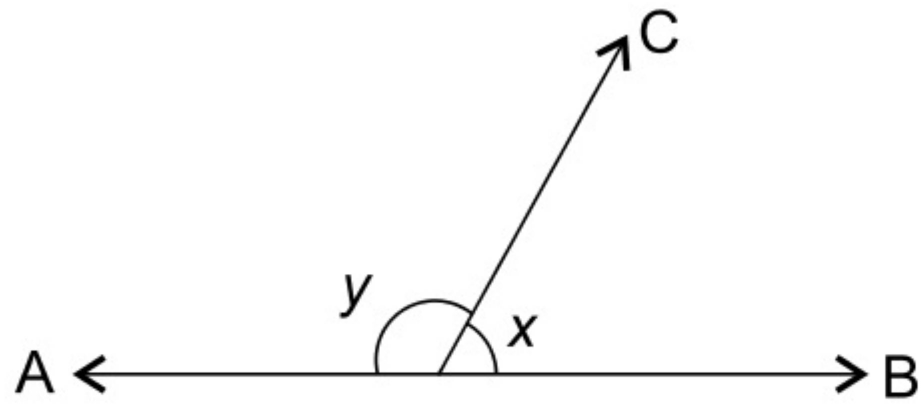
$AB = AC \neq BC$

$\angle B = \angle C \neq \angle A$



(xii) **Scalene Triangle**

$AB \neq BC \neq CA$



(xiii) **Linear Pair**

when $x + y = 180°$

Complementary Angles : Sum of two angles is 90° (like 30° and 60°, 10° and 80°)

Supplementary Angles : Sum of two angles is 180° (like 100° and 80°, 60° and 120°)

Two angles are called adjacent angles, if :

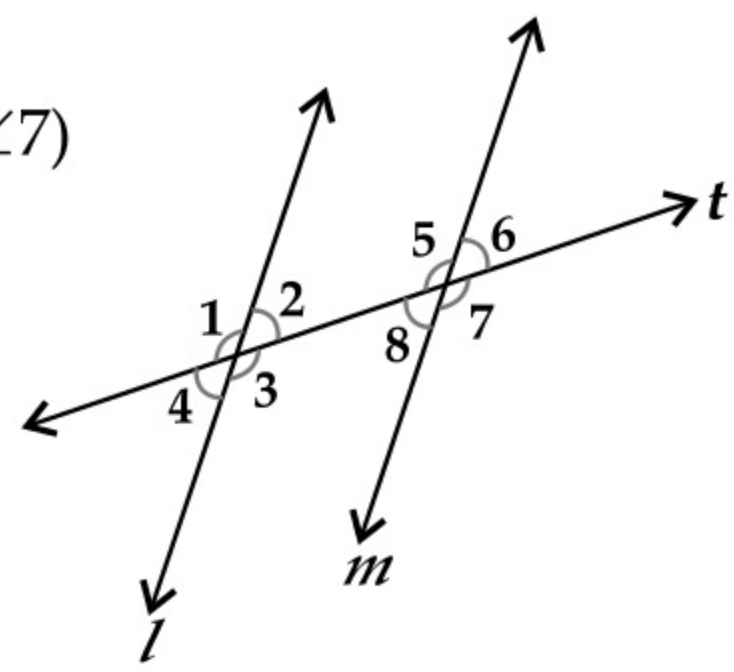
(i) they have the same vertex,

(ii) they have a common arm and

(iii) uncommon arms are on opposite side of the common arm.

Parallel Lines and Transversal :

- $l || m$ and t is transversal,
- **Corresponding Angles Pair :** $(\angle 1, \angle 5)$, $(\angle 4, \angle 8)$, $(\angle 2, \angle 6)$, $(\angle 3, \angle 7)$
  Property : They are equal ($\angle 1 = \angle 5$, $\angle 4 = \angle 8$ etc...)
- **Vertically Opposite Angles Pair :** $(\angle 1, \angle 3)$, $(\angle 2, \angle 4)$, $(\angle 6, \angle 8)$, $(\angle 5, \angle 7)$
  Property : They are equal ($\angle 1 = \angle 3$, $\angle 2 = \angle 4$ etc...)
- **Alternate Interior Angles Pair :** $(\angle 2, \angle 8)$, $(\angle 5, \angle 3)$
  Property : They are equal ($\angle 2 = \angle 8$, $\angle 5 = \angle 3$)
- **Alternate Exterior Angles Pair :** $(\angle 1, \angle 7)$, $(\angle 4, \angle 6)$
  Property : They are equal ($\angle 1 = \angle 7$, $\angle 4 = \angle 6$)
- **Consecutive Interior Angles :** $(\angle 2, \angle 5)$, $(\angle 3, \angle 8)$
  Property : They are supplementary ($\angle 2 + \angle 5 = 180°$, $\angle 3 + \angle 8 = 180°$)



Theorems and Proofs

Theorem 1

Statement : If two lines intersect each other, then the vertically opposite angles are equal.

Given : POQ and SOR are straight lines

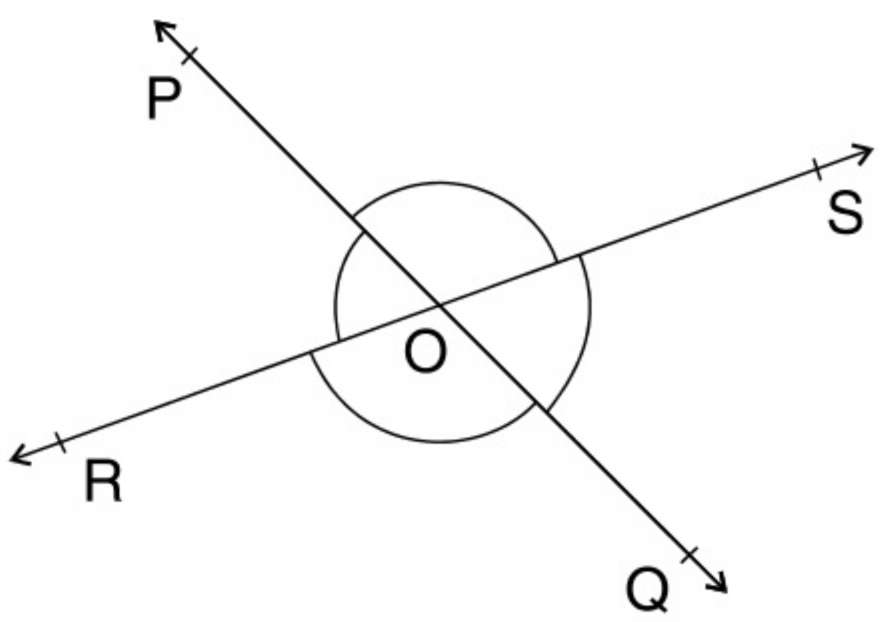
To Prove : $\angle POR = \angle SOQ$ and $\angle POS = \angle ROQ$

Proof :

In the given figure

$\angle POR + \angle POS = 180°$ ...(i) (Linear Pair)

$\angle SOQ + \angle POS = 180°$ ...(ii) (Linear Pair)



Comparing equation (i) and (ii), we get :

$\angle POR + \angle POS = \angle SOQ + \angle POS$

Therefore $\angle POR = \angle SOQ$ **Hence Proved.**

Similarly $\angle POS = \angle ROQ$

Theorem 2

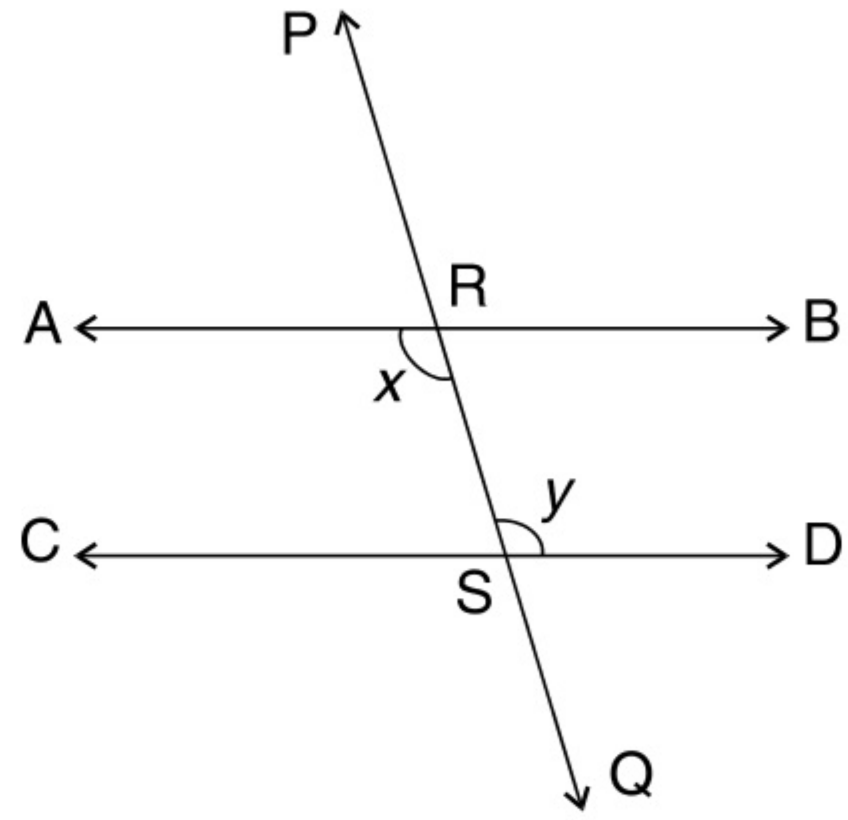
Statement : If a transversal intersects two parallel lines, then each pair of alternate interior angles are equal.

Given : $AB||CD$ and PQ is the transversal,

x and y are alternate interior angles

To Prove : $x = y$

Proof :



In the given figure

$\angle PRB = y$ ...(i) (Corresponding Angles)

$\angle PRB = x$ ...(ii) (Vertically Opposite Angles)

From equation (i) and (ii), we get

$x = y$, **Hence Proved.**

Theorem 3

Statement : If a transversal intersects two lines such that a pair of alternate interior angles are equal, then the two lines are parallel.

Theorem 4

Statement : If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.

Theorem 5

Statement : If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.

Theorem 6

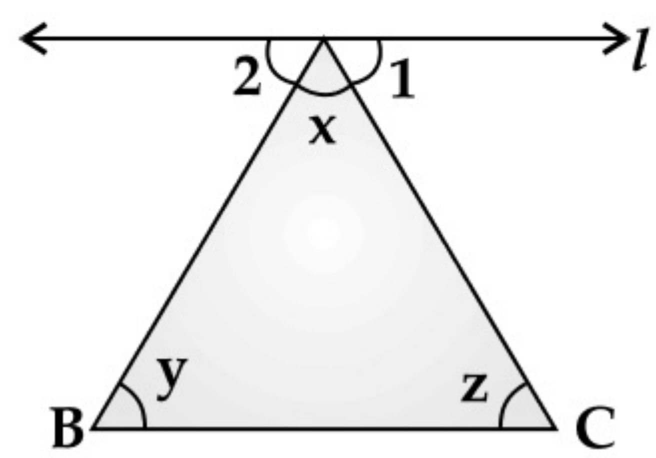
Statement : Lines which are parallel to the same line are parallel to each other.

Theorem 7

Statement : The sum of the angles of a triangle is 180°

To prove : Sum of all the angles of ΔABC is 180°.

Construction : Draw a line l parallel to BC.



Proof : Since $l||$BC, we have $\angle 2 = \angle y$ ...(i) (Alternate angles are equal)

Similarly

$\angle 1 = \angle z$ ...(ii) (Alternate angles are equal)

Also, sum of angles at a point A on line l is 180°.

$\therefore$ $\angle 2 + \angle x + \angle 1 = 180°$

i.e. $\angle y + \angle x + \angle z = 180°$ (from (i) and (ii))

$\therefore$ $\angle x + \angle y + \angle z = 180°$

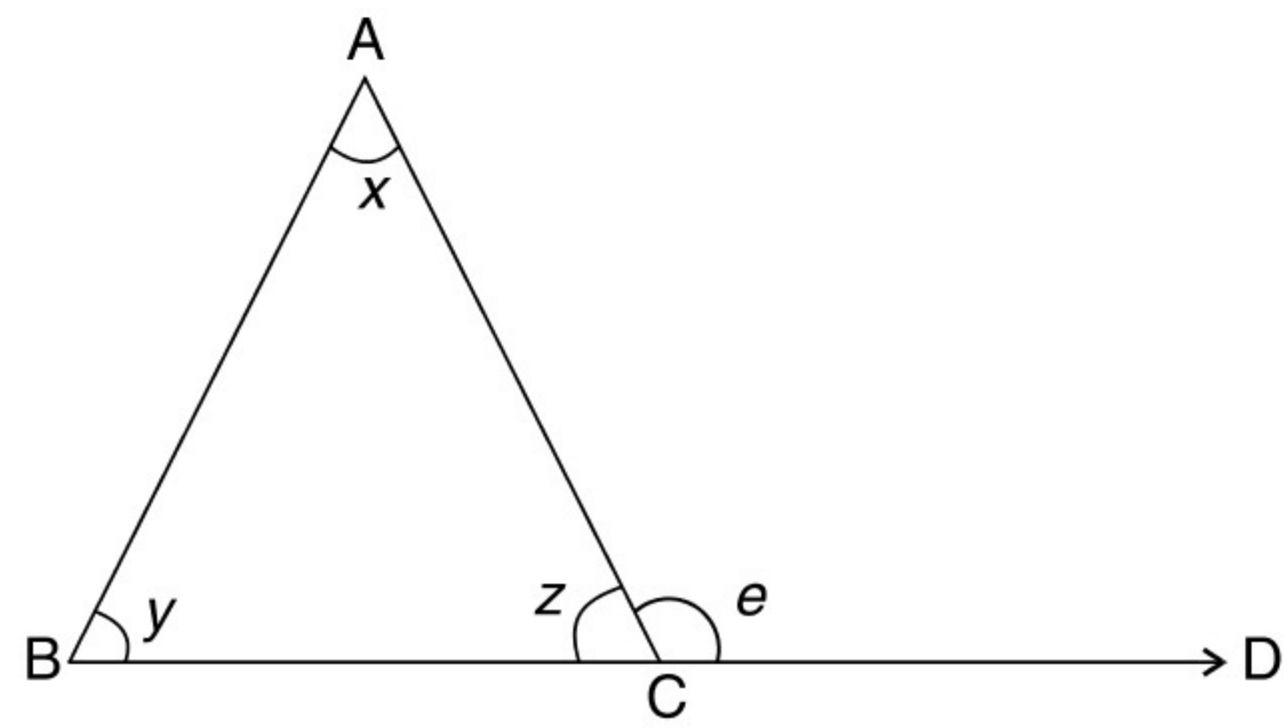
$\Rightarrow$ $\angle A + \angle B + \angle C = 180°$

Therefore,

sum of all the angles of Δ is 180°.

Theorem 8

Statement : If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.



Given : A triangle ABC with interior angles x, y and z, and exterior angle 'e'.

To Prove : $e = x + y$

Proof : In the figure, :

$x + y + z = 180°$ ...(i) (Angle Sum Property of Δ)

$e + z = 180°$ ...(ii) (Linear Pair)

Comparing equation (i) and (ii)

$x + y + z = e + z$

therefore, $x + y = e$. **Hence Proved.**