CHAPTER 7 : Triangles

Fundamentals :

- The geometrical figures of same shape and size are congruent to each other.
- Two circles of equal radii are congruent to each other.
- Two squares of equal edges are congruent.
- If two triangles ABC and PQR are congruent under the correspondence $A \leftrightarrow P$, $B \leftrightarrow Q$ and $C \leftrightarrow R$, then symbolically it is expressed as $\Delta ABC \cong \Delta PQR$.
- Two triangles are congruent if and only if their corresponding sides and the corresponding angles are equal.

  Congruence Criteria :

  - **<Axiom>*SAS* Congruence Rule :** Two triangles are congruent if any two sides and the included angle of one triangle are equal to the sides and the included angle of the other triangle.
  - **<Theorem>*AAS* Congruence Rule :** Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.

Theorems and Proofs(wherever required)

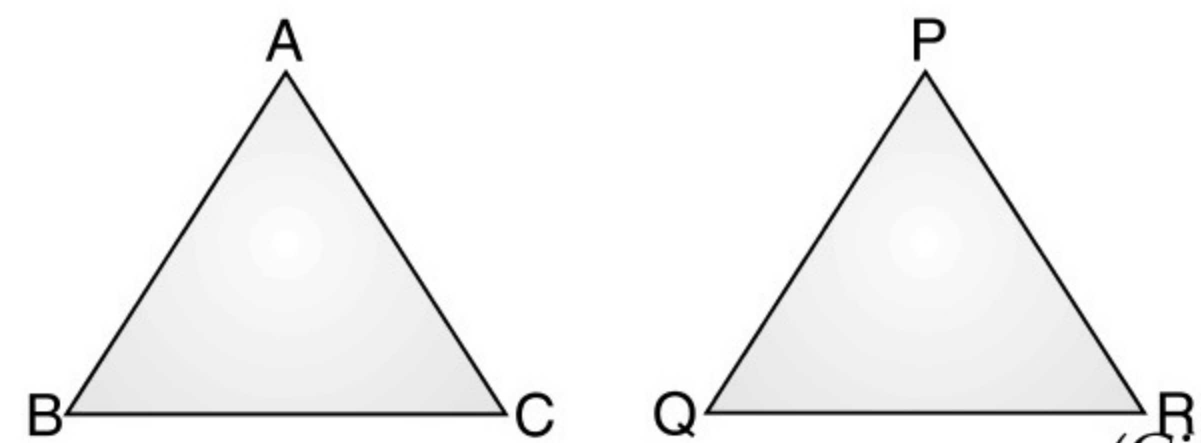
Theorem 1

Statement : *ASA* Congruence Rule : Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle.

Proof : We are given two triangles ABC and PQR in which :

$\angle B = \angle Q, \angle C = \angle R$

and $BC = QR$



We need to prove that $\Delta ABC \cong \Delta PQR$

There are three cases.

Case I : Let AB = PQ

In ΔABC and ΔPQR,

$\angle B = \angle Q$ (Given)

$BC = QR$ (Given)

$AB = PQ$ (Assumed)

$\Delta ABC \cong \angle PQR$ (By *SAS* rule)

Case II : Suppose $AB \neq PQ$ and $AB < PQ$

Take a point S on PQ such that $QS = AB$

Join RS.

In ΔABC and ΔSQR,

$AB = SQ$ (By construction)

$BC = QR$ (Given)

$\angle B = \angle Q$ (Given)

$\therefore$ $\Delta ABC \cong \angle SQR$ (By *SAS* rule)

$\angle ACB = \angle QRS$ (By c.p.c.t.)

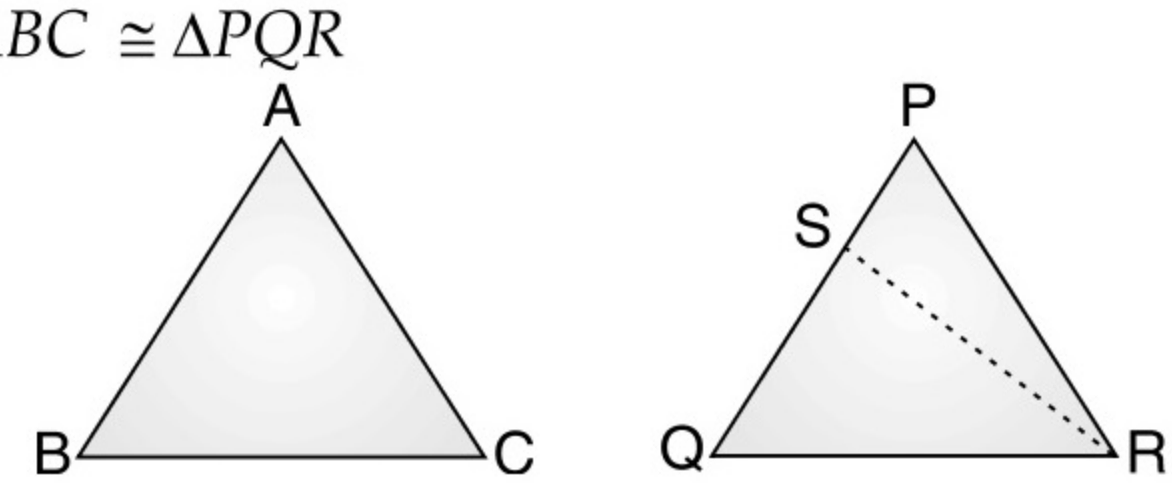
But $\angle QRP = \angle ACB$

$\Rightarrow$ $\angle QRP = \angle QRS$

which is impossible unless ray RS coincides with RP.

$\therefore$ AB must be equal to PQ.

So, $\Delta ABC \cong \Delta PQR$



Case III : If $AB > PQ$.

We can choose a point T on AB such that $TB = PQ$ and repeating the arguments as given in Case II, we can conclude that $AB = PQ$ and so,

$$\Delta ABC \cong \Delta PQR$$

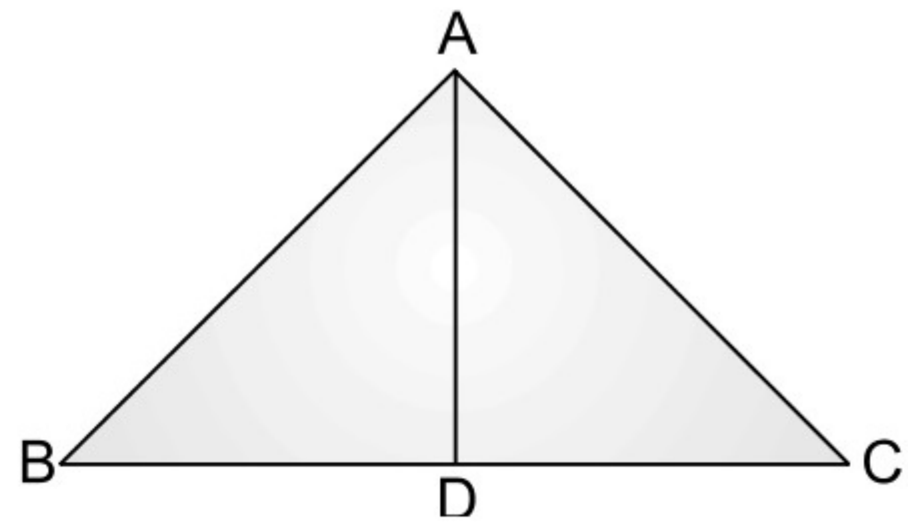
Theorem 2

Statement : Angles opposite to equal sides of an isosceles triangle are equal.

Given : ΔABC is an isosceles in which $AB = AC$

To prove : $\angle B = \angle C$

Construction : Draw the bisector of $\angle A$. Let D be the point of intersection of this bisector of $\angle A$ on BC.



Proof : In ΔBAD and ΔCAD,

	$BA = CA$	(Given)
	$\angle BAD = \angle CAD$	(By construction)
	$AD = AD$	(Common)
So,	$\Delta BAD \cong \Delta CAD$	(By *SAS* congruence)
	$\angle DBA = \angle DCA$	(By c.p.c.t.)
So,	$\angle B = \angle C$	**Hence Proved.**

Theorem 3

Statement : The sides opposite to equal angles of a triangle are equal.

Theorem 4

Statement : *SSS* Congruence Rule : If three sides of a triangle are equal to the three sides of another triangle, then the two triangles are congruent.

Theorem 5

Statement : *RHS* Congruence Rule : If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

Theorem 6

Statement : If two sides of a triangle are unequal, the angle opposite to the longer side is larger (or greater).

Theorem 7

Statement : In any triangle, the side opposite to the larger (greater) angle is longer.

Theorem 8

Statement : The sum of any two sides of a triangle is greater than third side.

Tips :

- In an isosceles triangle bisector of the vertical angle of a triangle bisects the base.
- The medians of an equilateral triangle are equal in length.
- Each angle of equilateral triangle is of $60°$.
- A point equidistant from two intersecting lines lies on the bisector of the angles formed by the two lines.
- In a triangle,
  - (i) angle opposite to the longer side is larger (greater).
  - (ii) side opposite to the larger (greater) angle is longer.
  - (iii) sum of any two sides is greater than the third side.
  - (iv) difference of any two sides of a triangle is less than the third side.