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## Chapter - 2

### (Polynomials)

Key Concepts

**Constants** : A symbol having a fixed numerical value is called a constant.

Example : 7, 3, -2,  $\frac{3}{7}$ , etc. are all constants.

**Variables** : A symbol which may be assigned different numerical values is known as variable.

Example :  $C = 2\pi r$       C - circumference of circle  
r - radius of circle

Where  $2$  &  $\pi$  are constants. while C and r are variable

**Algebraic expressions** : A combination of constants and variables connected by some or all of the operations +, -, X and  $\div$  is known as algebraic expression.

Example :  $4 + 9x - 5x^2y + \frac{3}{8}xy$  etc.

**Terms** : The several parts of an algebraic expression separated by '+' or '-' operations are called the terms of the expression.

Example :  $x^3 + 2x^2y + 4xy^2 + y^3 + 7$  is an algebraic expression containing 5 terms  $x^3, 2x^2y, 4xy^2, y^3$  & 7

**Polynomials** : An algebraic expression in which the variables involved have only non-negative integral powers is called a polynomial.

(i)  $5x^3 - 4x^2 - 6x - 3$  is a polynomial in variable x.

(ii)  $5 + 8x^{3/2} + 4x^{-2}$  is an expression but not a polynomial.

Polynomials are denoted by  $p(x)$ ,  $q(x)$  and  $r(x)$  etc.

**Coefficients** : In the polynomial  $x^3 + 3x^2 + 3x + 1$ , coefficient of  $x^3, x^2, x$  are 1, 3, 3 respectively and we also say that +1 is the constant term in it.

Degree of a polynomial in one variable : In case of a polynomial in one variable the highest power of the variable is called the degree of the polynomial.

Classification of polynomials on the basis of degree.

|     | Degree | Polynomial  | Example  |
|-----|--------|-------------|--|
| (a) | 1      | Linear      | $x + 1, 2x + 3$                                  |
| (b) | 2      | Quadratic   | $ax^2 + bx + c$ ( $a \neq 0$ ),<br>$x^2 + x + 1$ |
| (c) | 3      | Cubic       | $x^3 - 3x^2 + 1$                                 |
| (d) | 4      | Biquadratic | $x^4 - 1$  |

Classification of polynomials on the basis of no. of terms

|       | No. of terms | Polynomial | Examples                     |
|-------|--------------|------------|------------------------------|
| (i)   | 1            | Monomial   | $5, 3x, \frac{1}{3}y$ etc.   |
| (ii)  | 2            | Binomial   | $(3 + 6x), (x - 5y)$<br>etc. |
| (iii) | 3            | Trinomial  | $2x^2 + 4x + 2$              |

**Constant polynomial** : A polynomial containing one term only, consisting a constant term is called a constant polynomial the degree of non-zero constant polynomial is zero.

**Zero polynomial** : A polynomial consisting of one term, namely zero only is called a zero polynomial.

The degree of zero polynomial is not defined.

**Zeroes of a polynomial** : Let  $p(x)$  be a polynomial. If  $p(\alpha) = 0$ , then we say that  $\alpha$  is a zero of the polynomial of  $p(x)$ .

**Remark** : Finding the zeroes of polynomial  $p(x)$  means solving the equation  $p(x)=0$ .

**Remainder theorem** : Let  $f(x)$  be a polynomial of degree  $n \geq 1$  and let  $a$  be any real number. When  $f(x)$  is divided by  $(x - a)$  then the remainder is  $f(a)$

**Factor theorem** : Let  $f(x)$  be a polynomial of degree  $n > 1$  and let  $a$  be any real number.

(i) If  $f(a) = 0$  then  $(x - a)$  is factor of  $f(x)$

(ii) If  $(x - a)$  is a factor of  $f(x)$  then  $f(a) = 0$

**Factor** : A polynomial  $p(x)$  is called factor of  $q(x)$ , if  $p(x)$  divides  $q(x)$  exactly.

**Factorization** : To express a given polynomial as the product of polynomials each of degree less than that of the given polynomial such that no such a factor has a factor of lower degree, is called factorization.

$$\text{Example : } x^2 - 16 = (x + 4)(x - 4)$$

**Methods of Factorization :**

Factorization by taking out the common factor

e.g.

$$36q^3b - 60a^2bc = 12a^2b(3a - 5c)$$

Factorizing by grouping

$$\begin{aligned} ab + bc + ax + cx &= (ab + bc) + (ax + cx) \\ &= b(a + c) + x(a + c) \\ &= (a + c)(b + x) \end{aligned}$$

Factorization of quadratic trinomials by middle term splitting method.

$$\begin{aligned} x^2 + bx + c &= x^2 + (p + q)x + pq \\ &= (x + p)(x + q) \end{aligned}$$

**Identity** : Identity is a equation (trigonometric, algebraic ) which is true for every value of variable.

Some algebraic identities useful in factorization:

- (i)  $(x + y)^2 = x^2 + 2xy + y^2$
- (ii)  $(x - y)^2 = x^2 - 2xy + y^2$
- (iii)  $x^2 - y^2 = (x - y)(x + y)$
- (iv)  $(x + a)(x + b) = x^2 + (a + b)x + ab$
- (v)  $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
- (vi)  $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
- (vii)  $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
- (viii)  $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$   
 $x^3 + y^3 + z^3 = 3xyz \quad \text{if } x + y + z = 0$

(ix)  $x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$

(x)  $x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$

### Section - A

Q.1 Which of the following expressions is polynomial?

(i)  $x^5 - 2x^3 + x + 7$

(ii)  $y^3 - \sqrt{3}y$

(iii)  $5\sqrt{z} - 6$

(iv)  $x - \frac{1}{x}$

(v)  $x^{108} - 1$

(vi)  $\sqrt[3]{x} - 27$

(vii)  $x^{-2} + 2x^{-1} + 3$

Q.2 Write the degree of each of the following polynomial.

(i)  $2x - \sqrt{5}$

(ii)  $3 - x + x^2 - 6x^3$

(iii) 9

(iv)  $8x^4 - 36x + 5x^7$

(v)  $x^9 - x^5 + 3x^{10} + 8$

(vi)  $2 - 3x^2$

Q.3 (i) Give an example of a binomial of degree 27.

(ii) Give an example of a monomial of degree 16.

(iii) Give an example of trinomial of degree 3.

### Section -B

Q.4 If  $p(x) = 5 - 4x + 2x^2$  find (i)  $p(0)$  (ii)  $p(3)$  (iii)  $p(-2)$

Q.5 Find the zeros of the polynomials given below :

(i)  $p(x) = x - 5$

(ii)  $q(x) = x + 4$

(iii)  $h(x) = 6x - 1$

(iv)  $p(x) = ax + b$

(v)  $r(x) = x^2 + 3x$

(vi)  $l(x) = x^2 + 2x + 1$

Q.6 Find the remainder when  $f(x) = 12x^3 - 13x^2 - 5x + 7$  is divided by  $(3x + 2)$ ?

Q.7 Show that  $(x + 5)$  is a factor of the polynomial

$$f(x) = x^3 + x^2 + 3x + 115$$

Q.8 Find the value of a for which  $(x - a)$  is a factor of the polynomial.

$$f(x) = x^5 - a^2x^3 + 2x + a - 3$$

## Section - C

Q.9 Factorize the following expressions.

(i)  $5x^2 - 20xy$

(ii)  $5a(b + c) - 7b(b + c)$

(iii)  $x(x - y)^2 + 3x^2y(x - y)$

(iv)  $6ab - b^2 + 12ac - 2bc$

(v)  $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$

Q.10 Factorize :

(i)  $9x^2 - 16y^2$

(ii)  $x^3 - x$

Q.11 Factorize:

$a(a - 1) - b(b - 1)$

Q.12 Factorize following expressions.

(i)  $x^2 + 9x + 18$

(ii)  $x^2 - 4x - 2$

(iii)  $x^2 - 9x + 18$

(iv)  $x^2 - 19x + 78$

Q.13 Calculate  $(997)^2$  using algebraic identities.

Q.14 Calculate  $103 \times 107$  using algebraic identities.

Q.15 Expand  $(2a + 3b + 4c)^2$ .

Q.16 Factorize  $4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$ .

Q.17 Expand (i)  $(4a + 5b)^3$       (ii)  $(5x - 3y)^3$

Q.18 Evaluate (i)  $(95)^3$       (ii)  $(106)^3$

Q.19 Factorize      (i)  $x^3 + 64$       (ii)  $27x^3 + 125y^3$

(iii)  $8a^3 - 27b^3$       (iv)  $1 - 64a^3$

### Section - D

Q.20 Factorize

$$a^3 + 27b^3 + 8c^3 - 18abc$$

Q.21 Factorize

$$(p - q)^3 + (q - r)^3 + (r - p)^3$$

Q.22 Find the product

$$(3x - 5y - 4)(9x^2 + 25y^2 + 15xy + 12x - 20y + 16)$$

Q.23 If  $x + y + z = 9$  and  $xy + yz + zx = 23$  then find the value of

$$(x^3 + y^3 + z^3 - 3xyz)?$$

### Self Evaluation

Q.24 Which of the following expression is a polynomial?

(a)  $\sqrt{x} - 1$       (b)  $\frac{x-1}{x+1}$       (c)  $x^2 - \frac{2}{x^2} + 5$  (d)  $x^2 + \frac{2x^{3/2}}{\sqrt{x}} + 6$

Q.25 Degree of zero polynomial is

(a) 1      (b) 0      (c) not defined      (d) none of these

Q.26 For what value of k is the polynomial  $p(x) = 2x^3 - kx^2 + 3x + 10$  exactly divisible by  $(x + 2)$ ?

(a)  $\frac{-1}{3}$       (b)  $\frac{1}{3}$       (c) 3      (d) -3

Q.27 The zeroes of the polynomial  $p(x) = 3x^2 - 1$  are

(a)  $\frac{1}{3}$       (b)  $\frac{1}{\sqrt{3}}$       (c)  $\frac{-1}{\sqrt{3}}$       (d)  $\frac{1}{\sqrt{3}}$  and  $\frac{-1}{\sqrt{3}}$

Q.28 If  $\frac{x}{y} + \frac{y}{x} = -1$  where  $x \neq 0, y \neq 0$  then find the value of  $x^3 - y^3$ .

Q.29 If  $(x + 2)$  and  $(x - 1)$  are factors of  $(x^3 + 10x^2 + mx + n)$  then find value of m & n?

Q.30 Find the value of  $(369)^2 - (368)^2$

Q.31 Find value of  $104 \times 96$

Q.32 If  $a + b + c = 0$  find value of  $\left(\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right)$  ?

## Answers

Q.1 (i), (ii), (v)

Q.2 (i) 1 (ii) 3 (iii) 0 (iv) 4 (v) 9 (vi) 2

Q.4 (i)  $p(0) = 5$  (ii)  $p(3) = 11$  (iv) 21

Q.5 (i)  $x = 5$  (ii)  $x = -4$  (iii)  $x = 1/6$  (iv)  $x = -b/a$

(v)  $x = 0, x = -3$  (vi)  $x = -1, -1$

Q.6 remainder = 1

Q.8  $a = 1$

Q.9 (i)  $5x(x - 4y)$

(ii)  $(b + c)(5a - 7b)$

(iii)  $x(x - y)[(x - y) + 3xy]$

(iv)  $(b + 2c)(6a - b)$

(v)  $\left(x + \frac{1}{x}\right)\left(x + \frac{1}{x} - 2\right)$

Q.10 (i)  $(3x + 4y)(3x - 4y)$  (ii)  $x(x + 1)(x - 1)$

Q.11  $(a-b)(a+b-1)$

Q.12 (i)  $(x + 6)(x + 3)$

(ii)  $(x - 7)(x + 3)$

(iii)  $(x - 6)(x - 3)$

(iv)  $(x - 6)(x - 13)$

Q.13 994009

Q.14 11021

Q.15  $4a^2 + 9b^2 + 16c^2 + 12ab + 24bc + 16ac$

Q.16  $(2x - y + z)^2$

Q.17 (i)  $64a^3 + 125b^3 + 240a^2b + 300ab^2$

(ii)  $125x^3 - 27y^3 - 225x^2y + 135xy^2$

- Q.18 (i) 857375 (ii) 1191016
- Q.19 (i)  $(x + 4)(x^2 - 4x + 16)$   
(ii)  $(3x+5y)(9x^2-15xy+25y^2)$   
(iii)  $(2a - 3b)(4a^2 + 6ab + 9b^2)$   
(iv)  $(1 - 4a)(1 + 4a + 16a^2)$
- Q.20  $(a + 3b + 2c)(a^2 + 9b^2 + 4c^2 - 3ab - 6bc - 2ac)$
- Q.21  $3(p - q)(q - r)(r - p)$
- Q.22  $27x^3 - 125y^3 - 64 - 180xy.$
- Q.23 108
- Q.24 (d)  $x^2 + \frac{2x^{\frac{3}{2}}}{x^{\frac{1}{2}}} + 6$
- Q.25 (c) not defined
- Q.26 (d) -3
- Q.27 (d)  $\frac{1}{\sqrt{3}} \& \frac{-1}{\sqrt{3}}$
- Q.28 0
- Q.29  $m = 7, n = -18$
- Q.30 737
- Q.31 9984
- Q.32 3
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