

STATISTICS AND PROBABILITY

(A) Main Concepts and Results

Statistics

Measures of Central Tendency

(a) Mean of Grouped Data

- (i) To find the mean of grouped data, it is assumed that the frequency of each class interval is centred around its mid-point.
- (ii) **Direct Method**

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i},$$

where the x_i (class mark) is the mid-point of the i th class interval and f_i is the corresponding frequency.

- (iii) **Assumed Mean Method**

$$\text{Mean } (\bar{x}) = a + \frac{\sum f_i d_i}{\sum f_i},$$

a is the *assumed mean* and $d_i = x_i - a$ are the deviations of x_i from a for each i .

(iv) Step-deviation Method

$$\text{Mean } (\bar{x}) = a + h \left(\frac{\sum f_i u_i}{\sum f_i} \right),$$

where a is the assumed mean, h is the class size and $u_i = \frac{x_i - a}{h}$.

- (v) If the class sizes are unequal, the formula in (iv) can still be applied by taking h to be a suitable divisor of all the d_i 's.

(b) Mode of Grouped Data

- (i) In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies. To find the mode of grouped data, locate the class with the maximum frequency. This class is known as the *modal class*. The mode of the data is a value inside the modal class.
- (ii) Mode of the grouped data can be calculated by using the formula

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h,$$

where l is the lower limit of the modal class, h is the size of the class, f_1 is frequency of the modal class and f_0 and f_2 are the frequencies of the classes preceding and succeeding the modal class, respectively.

(c) Median of Grouped Data

- (i) Cumulative frequency table – the less than type and the more than type of the grouped frequency distribution.
- (ii) If n is the total number of observations, locate the class whose cumulative frequency is greater than (and nearest to) $\frac{n}{2}$. This class is called the *median class*.
- (iii) Median of the grouped data can be calculated by using the formula :

$$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f} \right) h,$$

where l is the lower limit of the median class, n is the number of observations, h is the class size, cf is the cumulative frequency of the class preceding the median class and f is the frequency of the median class.

(d) Graphical Representation of Cumulative Frequency Distribution (Ogive)

– Less than type and more than type.

- (i) To find median from the graph of cumulative frequency distribution (less than type) of a grouped data.
- (ii) To find median from the graphs of cumulative frequency distributions (of less than type and more than type) as the abscissa of the point of intersection of the graphs.

Probability

- Random experiment, outcome of an experiment, event, elementary events.
- Equally likely outcomes.
- The theoretical (or classical) probability of an event E [denoted by $P(E)$] is given by

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Number of all possible outcomes of the experiment}}$$

where the outcomes of the experiment are *equally likely*.

- The probability of an event can be any number between 0 and 1. It can also be 0 or 1 in some special cases.
- The sum of the probabilities of all the elementary events of an experiment is 1.
- For an event E , $P(E) + P(\bar{E}) = 1$,
where \bar{E} is the event 'not E '. \bar{E} is called the complement of the event E .
- Impossible event, sure or a certain event

(B) Multiple Choice Questions

Choose the correct answer from the given four options:

Sample Question 1 : Construction of a cumulative frequency table is useful in determining the

- | | |
|----------|----------------------------------|
| (A) mean | (B) median |
| (C) mode | (D) all the above three measures |

Solution : Answer (B)

Sample Question 2 : In the following distribution :

| Monthly income range (in Rs) | Number of families |
|------------------------------|--------------------|
| Income more than Rs 10000 | 100 |
| Income more than Rs 13000 | 85 |
| Income more than Rs 16000 | 69 |
| Income more than Rs 19000 | 50 |
| Income more than Rs 22000 | 33 |
| Income more than Rs 25000 | 15 |

the number of families having income range (in Rs) 16000 – 19000 is

- (A) 15 (B) 16 (C) 17 (D) 19

Solution : Answer (D)

Sample Question 3 : Consider the following frequency distribution of the heights of 60 students of a class :

| Height (in cm) | Number of students |
|----------------|--------------------|
| 150-155 | 15 |
| 155-160 | 13 |
| 160-165 | 10 |
| 165-170 | 8 |
| 170-175 | 9 |
| 175-180 | 5 |

The sum of the lower limit of the modal class and upper limit of the median class is

- (A) 310 (B) 315 (C) 320 (D) 330

Solution : Answer (B)

Sample Question 4 : Which of the the following can be the probability of an event?

- (A) -0.04 (B) 1.004 (C) $\frac{18}{23}$ (D) $\frac{8}{7}$

Solution : Answer (C)

Sample Question 5 : A card is selected at random from a well shuffled deck of 52 playing cards. The probability of its being a face card is

- (A) $\frac{3}{13}$ (B) $\frac{4}{13}$ (C) $\frac{6}{13}$ (D) $\frac{9}{13}$

Solution : Answer (A)

Sample Question 6 : A bag contains 3 red balls, 5 white balls and 7 black balls. What is the probability that a ball drawn from the bag at random will be neither red nor black?

- (A) $\frac{1}{5}$ (B) $\frac{1}{3}$ (C) $\frac{7}{15}$ (D) $\frac{8}{15}$

Solution : Answer (B)

EXERCISE 13.1

Choose the correct answer from the given four options:

1. In the formula

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i},$$

for finding the mean of grouped data d_i 's are deviations from a of

- (A) lower limits of the classes
 (B) upper limits of the classes
 (C) mid points of the classes
 (D) frequencies of the class marks
2. While computing mean of grouped data, we assume that the frequencies are
 (A) evenly distributed over all the classes
 (B) centred at the classmarks of the classes
 (C) centred at the upper limits of the classes
 (D) centred at the lower limits of the classes
3. If x_i 's are the mid points of the class intervals of grouped data, f_i 's are the corresponding frequencies and \bar{x} is the mean, then $\sum (f_i x_i - \bar{x})$ is equal to
 (A) 0 (B) -1 (C) 1 (D) 2
4. In the formula $\bar{x} = a + h \left(\frac{\sum f_i u_i}{\sum f_i} \right)$, for finding the mean of grouped frequency

distribution, $u_i =$

- (A) $\frac{x_i + a}{h}$ (B) $h(x_i - a)$ (C) $\frac{x_i - a}{h}$ (D) $\frac{a - x_i}{h}$

5. The abscissa of the point of intersection of the less than type and of the more than type cumulative frequency curves of a grouped data gives its
 (A) mean (B) median
 (C) mode (D) all the three above
6. For the following distribution :

| | | | | | |
|------------------|-----|------|-------|-------|-------|
| Class | 0-5 | 5-10 | 10-15 | 15-20 | 20-25 |
| Frequency | 10 | 15 | 12 | 20 | 9 |

the sum of lower limits of the median class and modal class is

- (A) 15 (B) 25 (C) 30 (D) 35
7. Consider the following frequency distribution :

| | | | | | |
|------------------|-----|------|-------|-------|-------|
| Class | 0-5 | 6-11 | 12-17 | 18-23 | 24-29 |
| Frequency | 13 | 10 | 15 | 8 | 11 |

The upper limit of the median class is

- (A) 17 (B) 17.5 (C) 18 (D) 18.5
8. For the following distribution :

| Marks | Number of students |
|--------------|---------------------------|
| Below 10 | 3 |
| Below 20 | 12 |
| Below 30 | 27 |
| Below 40 | 57 |
| Below 50 | 75 |
| Below 60 | 80 |

the modal class is

- (A) 10-20 (B) 20-30 (C) 30-40 (D) 50-60
9. Consider the data :

| | | | | | | | |
|------------------|-------|--------|---------|---------|---------|---------|---------|
| Class | 65-85 | 85-105 | 105-125 | 125-145 | 145-165 | 165-185 | 185-205 |
| Frequency | 4 | 5 | 13 | 20 | 14 | 7 | 4 |

The difference of the upper limit of the median class and the lower limit of the modal class is

- (A) 0 (B) 19 (C) 20 (D) 38

10. The times, in seconds, taken by 150 athletes to run a 110 m hurdle race are tabulated below :

| | | | | | | |
|------------------|---------|---------|-----------|-----------|-----------|---------|
| Class | 13.8-14 | 14-14.2 | 14.2-14.4 | 14.4-14.6 | 14.6-14.8 | 14.8-15 |
| Frequency | 2 | 4 | 5 | 71 | 48 | 20 |

The number of athletes who completed the race in less than 14.6 seconds is :

- (A) 11 (B) 71 (C) 82 (D) 130

11. Consider the following distribution :

| Marks obtained | Number of students |
|--------------------------|---------------------------|
| More than or equal to 0 | 63 |
| More than or equal to 10 | 58 |
| More than or equal to 20 | 55 |
| More than or equal to 30 | 51 |
| More than or equal to 40 | 48 |
| More than or equal to 50 | 42 |

the frequency of the class 30-40 is

- (A) 3 (B) 4 (C) 48 (D) 51

12. If an event cannot occur, then its probability is

- (A) 1 (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ (D) 0

13. Which of the following cannot be the probability of an event?

- (A) $\frac{1}{3}$ (B) 0.1 (C) 3% (D) $\frac{17}{16}$

14. An event is very unlikely to happen. Its probability is closest to

- (A) 0.0001 (B) 0.001 (C) 0.01 (D) 0.1

15. If the probability of an event is p , the probability of its complementary event will be

- (A) $p - 1$ (B) p (C) $1 - p$ (D) $1 - \frac{1}{p}$

16. The probability expressed as a percentage of a particular occurrence can never be
(A) less than 100 (B) less than 0
(C) greater than 1 (D) anything but a whole number
17. If $P(A)$ denotes the probability of an event A , then
(A) $P(A) < 0$ (B) $P(A) > 1$ (C) $0 \leq P(A) \leq 1$ (D) $-1 \leq P(A) \leq 1$
18. A card is selected from a deck of 52 cards. The probability of its being a red face card is
(A) $\frac{3}{26}$ (B) $\frac{3}{13}$ (C) $\frac{2}{13}$ (D) $\frac{1}{2}$
19. The probability that a non leap year selected at random will contain 53 sundays is
(A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{3}{7}$ (D) $\frac{5}{7}$
20. When a die is thrown, the probability of getting an odd number less than 3 is
(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 0
21. A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is
(A) 4 (B) 13 (C) 48 (D) 51
22. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is
(A) 7 (B) 14 (C) 21 (D) 28
23. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought?
(A) 40 (B) 240 (C) 480 (D) 750
24. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 5 is
(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $\frac{1}{3}$
25. Someone is asked to take a number from 1 to 100. The probability that it is a prime is
(A) $\frac{1}{5}$ (B) $\frac{6}{25}$ (C) $\frac{1}{4}$ (D) $\frac{13}{50}$

26. A school has five houses A, B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and rest from house E. A single student is selected at random to be the class monitor. The probability that the selected student is not from A, B and C is

- (A) $\frac{4}{23}$ (B) $\frac{6}{23}$ (C) $\frac{8}{23}$ (D) $\frac{17}{23}$

(C) Short Answer Questions with Reasoning

Sample Question 1 : The mean of ungrouped data and the mean calculated when the same data is grouped are always the same. Do you agree with this statement? Give reason for your answer.

Solution : The statement is not true. The reason is that when we calculated mean of a grouped data, it is assumed that frequency of each class is centred at the mid-point of the class. Because of this, two values of the mean, namely, those from ungrouped and grouped data are rarely the same.

Sample Question 2 : Is it correct to say that an ogive is a graphical representation of a frequency distribution? Give reason.

Solution : Graphical representation of a frequency distribution may not be an ogive. It may be a histogram. An ogive is a graphical representation of cumulative frequency distribution.

Sample Question 3 : In any situation that has only two possible outcomes, each outcome will have probability $\frac{1}{2}$. True or false? Why?

Solution : False, because the probability of each outcome will be $\frac{1}{2}$ only when the two outcomes are equally likely otherwise not.

EXERCISE 13.2

- The median of an ungrouped data and the median calculated when the same data is grouped are always the same. Do you think that this is a correct statement? Give reason.
- In calculating the mean of grouped data, grouped in classes of equal width, we may use the formula

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

where a is the assumed mean. a must be one of the mid-points of the classes. Is the last statement correct? Justify your answer.

3. Is it true to say that the mean, mode and median of grouped data will always be different? Justify your answer.
4. Will the median class and modal class of grouped data always be different? Justify your answer.
5. In a family having three children, there may be no girl, one girl, two girls or three girls. So, the probability of each is $\frac{1}{4}$. Is this correct? Justify your answer.
6. A game consists of spinning an arrow which comes to rest pointing at one of the regions (1, 2 or 3) (Fig. 13.1). Are the outcomes 1, 2 and 3 equally likely to occur? Give reasons.

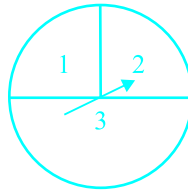


Fig. 13.1

7. Apoorv throws two dice once and computes the product of the numbers appearing on the dice. Peehu throws one die and squares the number that appears on it. Who has the better chance of getting the number 36? Why?
8. When we toss a coin, there are two possible outcomes - Head or Tail. Therefore, the probability of each outcome is $\frac{1}{2}$. Justify your answer.
9. A student says that if you throw a die, it will show up 1 or not 1. Therefore, the probability of getting 1 and the probability of getting 'not 1' each is equal to $\frac{1}{2}$. Is this correct? Give reasons.
10. I toss three coins together. The possible outcomes are no heads, 1 head, 2 heads and 3 heads. So, I say that probability of no heads is $\frac{1}{4}$. What is wrong with this conclusion?
11. If you toss a coin 6 times and it comes down heads on each occasion. Can you say that the probability of getting a head is 1? Give reasons.

12. Sushma tosses a coin 3 times and gets tail each time. Do you think that the outcome of next toss will be a tail? Give reasons.
13. If I toss a coin 3 times and get head each time, should I expect a tail to have a higher chance in the 4th toss? Give reason in support of your answer.
14. A bag contains slips numbered from 1 to 100. If Fatima chooses a slip at random from the bag, it will either be an odd number or an even number. Since this situation has only two possible outcomes, so, the probability of each is $\frac{1}{2}$. Justify.

(D) Short Answer Questions

Sample Question 1 : Construct the cumulative frequency distribution of the following distribution :

| Class | 12.5-17.5 | 17.5-22.5 | 22.5-27.5 | 27.5-32.5 | 32.5-37.5 |
|-----------|-----------|-----------|-----------|-----------|-----------|
| Frequency | 2 | 22 | 19 | 14 | 13 |

Solution : The required cumulative frequency distribution of the given distribution is given below :

| Class | Frequency | Cumulative frequency |
|-----------|-----------|----------------------|
| 12.5-17.5 | 2 | 2 |
| 17.5-22.5 | 22 | 24 |
| 22.5-27.5 | 19 | 43 |
| 27.5-32.5 | 14 | 57 |
| 32.5-37.5 | 13 | 70 |

Sample Question 2 : Daily wages of 110 workers, obtained in a survey, are tabulated below :

| Daily wages (in Rs) | Number of workers |
|---------------------|-------------------|
| 100-120 | 10 |
| 120-140 | 15 |
| 140-160 | 20 |
| 160-180 | 22 |
| 180-200 | 18 |
| 200-220 | 12 |
| 220-240 | 13 |

Compute the mean daily wages of these workers.

Solution : We first find the classmark, x_i , of each class and then proceed as follows:

| Daily wages (in Rs) Classes | Class marks (x_i) | Number of workers (f_i) | $f_i x_i$ |
|-----------------------------------|--------------------------|--------------------------------|-----------|
| 100-120 | 110 | 10 | 1100 |
| 120-140 | 130 | 15 | 1950 |
| 140-160 | 150 | 20 | 3000 |
| 160-180 | 170 | 22 | 3740 |
| 180-200 | 190 | 18 | 3420 |
| 200-220 | 210 | 12 | 2520 |
| 220-240 | 230 | 13 | 2990 |

$$\sum f_i = 110,$$

$$\sum f_i x_i = 18720$$

$$\text{Therefore, Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{18720}{110} = 170.20$$

Hence, the mean daily wages of the workers is Rs 170.20.

Note : Mean daily wages can also be calculated by the assumed mean method or step deviation method.

Sample Question 3 : The percentage of marks obtained by 100 students in an examination are given below:

| Marks | 30-35 | 35-40 | 40-45 | 45-50 | 50-55 | 55-60 | 60-65 |
|-----------|-------|-------|-------|-------|-------|-------|-------|
| Frequency | 14 | 16 | 18 | 23 | 18 | 8 | 3 |

Determine the median percentage of marks.

Solution :

| Marks (Class) | Number of Students (Frequency) | Cumulative frequency |
|------------------|-----------------------------------|----------------------|
| 30-35 | 14 | 14 |
| 35-40 | 16 | 30 |
| 40-45 | 18 | 48 |
| 45-50 | 23 | 71 ← Median class |
| 50-55 | 18 | 89 |
| 55-60 | 8 | 97 |
| 60-65 | 3 | 100 |

Here, $n = 100$.

Therefore, $\frac{n}{2} = 50$, This observation lies in the class 45-50.

l (the lower limit of the median class) = 45

cf (the cumulative frequency of the class preceding the median class) = 48

f (the frequency of the median class) = 23

h (the class size) = 5

$$\begin{aligned} \text{Median} &= l + \left(\frac{\frac{n}{2} - \text{cf}}{f} \right) h \\ &= 45 + \left(\frac{50 - 48}{23} \right) \times 5 \\ &= 45 + \frac{10}{23} = 45.4 \end{aligned}$$

So, the median percentage of marks is 45.4.

Sample Question 4 : The frequency distribution table of agricultural holdings in a village is given below :

| | | | | | | |
|---------------------------------------|-----|-----|-----|-----|------|-------|
| Area of land (in hectares) | 1-3 | 3-5 | 5-7 | 7-9 | 9-11 | 11-13 |
| Number of families | 20 | 45 | 80 | 55 | 40 | 12 |

Find the modal agricultural holdings of the village.

Solution : Here the maximum class frequency is 80, and the class corresponding to this frequency is 5-7.

So, the modal class is 5-7.

l (lower limit of modal class) = 5

f_i (frequency of the modal class) = 80

f_0 (frequency of the class preceding the modal class) = 45
 f_2 (frequency of the class succeeding the modal class) = 55
 h (class size) = 2

$$\begin{aligned}
 \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
 &= 5 + \left(\frac{80 - 45}{2(80) - 45 - 55} \right) \times 2 \\
 &= 5 + \frac{35}{60} \times 2 = 5 + \frac{35}{30} \\
 &= 5 + 1.2 = 6.2
 \end{aligned}$$

Hence, the modal agricultural holdings of the village is 6.2 hectares.

EXERCISE 13.3

1. Find the mean of the distribution :

| | | | | |
|------------------|-----|-----|-----|------|
| Class | 1-3 | 3-5 | 5-7 | 7-10 |
| Frequency | 9 | 22 | 27 | 17 |

2. Calculate the mean of the scores of 20 students in a mathematics test :

| | | | | | |
|---------------------------|-------|-------|-------|-------|-------|
| Marks | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
| Number of students | 2 | 4 | 7 | 6 | 1 |

3. Calculate the mean of the following data :

| | | | | |
|------------------|-------|--------|---------|---------|
| Class | 4 - 7 | 8 - 11 | 12 - 15 | 16 - 19 |
| Frequency | 5 | 4 | 9 | 10 |

4. The following table gives the number of pages written by Sarika for completing her own book for 30 days :

| | | | | | |
|--|-------|-------|-------|-------|-------|
| Number of pages written per day | 16-18 | 19-21 | 22-24 | 25-27 | 28-30 |
| Number of days | 1 | 3 | 4 | 9 | 13 |

Find the mean number of pages written per day.

5. The daily income of a sample of 50 employees are tabulated as follows :

| | | | | |
|----------------------------|-------|---------|---------|---------|
| Income (in Rs) | 1-200 | 201-400 | 401-600 | 601-800 |
| Number of employees | 14 | 15 | 14 | 7 |

Find the mean daily income of employees.

6. An aircraft has 120 passenger seats. The number of seats occupied during 100 flights is given in the following table :

| | | | | | |
|------------------------|---------|---------|---------|---------|---------|
| Number of seats | 100-104 | 104-108 | 108-112 | 112-116 | 116-120 |
| Frequency | 15 | 20 | 32 | 18 | 15 |

Determine the mean number of seats occupied over the flights.

7. The weights (in kg) of 50 wrestlers are recorded in the following table :

| | | | | | |
|----------------------------|---------|---------|---------|---------|---------|
| Weight (in kg) | 100-110 | 110-120 | 120-130 | 130-140 | 140-150 |
| Number of wrestlers | 4 | 14 | 21 | 8 | 3 |

Find the mean weight of the wrestlers.

8. The mileage (km per litre) of 50 cars of the same model was tested by a manufacturer and details are tabulated as given below :

| | | | | |
|-----------------------|-------|-------|-------|-------|
| Mileage (km/l) | 10-12 | 12-14 | 14-16 | 16-18 |
| Number of cars | 7 | 12 | 18 | 13 |

Find the mean mileage.

The manufacturer claimed that the mileage of the model was 16 km/litre. Do you agree with this claim?

9. The following is the distribution of weights (in kg) of 40 persons :

| | | | | | | | | |
|--------------------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Weight (in kg) | 40-45 | 45-50 | 50-55 | 55-60 | 60-65 | 65-70 | 70-75 | 75-80 |
| Number of persons | 4 | 4 | 13 | 5 | 6 | 5 | 2 | 1 |

Construct a cumulative frequency distribution (of the less than type) table for the data above.

10. The following table shows the cumulative frequency distribution of marks of 800 students in an examination:

| Marks | Number of students |
|--------------|---------------------------|
| Below 10 | 10 |
| Below 20 | 50 |
| Below 30 | 130 |
| Below 40 | 270 |
| Below 50 | 440 |
| Below 60 | 570 |
| Below 70 | 670 |
| Below 80 | 740 |
| Below 90 | 780 |
| Below 100 | 800 |

Construct a frequency distribution table for the data above.

11. Form the frequency distribution table from the following data :

| Marks (out of 90) | Number of candidates |
|--------------------------|----------------------|
| More than or equal to 80 | 4 |
| More than or equal to 70 | 6 |
| More than or equal to 60 | 11 |
| More than or equal to 50 | 17 |
| More than or equal to 40 | 23 |
| More than or equal to 30 | 27 |
| More than or equal to 20 | 30 |
| More than or equal to 10 | 32 |
| More than or equal to 0 | 34 |

12. Find the unknown entries a, b, c, d, e, f in the following distribution of heights of students in a class :

| Height (in cm) | Frequency | Cumulative frequency |
|----------------|-----------|----------------------|
| 150-155 | 12 | a |
| 155-160 | b | 25 |
| 160-165 | 10 | c |
| 165-170 | d | 43 |
| 170-175 | e | 48 |
| 175-180 | 2 | f |
| Total | 50 | |

13. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day :

| Age (in years) | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 |
|--------------------|-------|-------|-------|-------|-------|-------|
| Number of patients | 60 | 42 | 55 | 70 | 53 | 20 |

Form:

- (i) Less than type cumulative frequency distribution.
- (ii) More than type cumulative frequency distribution.

14. Given below is a cumulative frequency distribution showing the marks secured by 50 students of a class :

| Marks | Below 20 | Below 40 | Below 60 | Below 80 | Below 100 |
|--------------------|----------|----------|----------|----------|-----------|
| Number of students | 17 | 22 | 29 | 37 | 50 |

Form the frequency distribution table for the data.

15. Weekly income of 600 families is tabulated below :

| Weekly income (in Rs) | Number of families |
|-----------------------|--------------------|
| 0-1000 | 250 |
| 1000-2000 | 190 |
| 2000-3000 | 100 |
| 3000-4000 | 40 |
| 4000-5000 | 15 |
| 5000-6000 | 5 |
| Total | 600 |

Compute the median income.

16. The maximum bowling speeds, in km per hour, of 33 players at a cricket coaching centre are given as follows :

| Speed (km/h) | 85-100 | 100-115 | 115-130 | 130-145 |
|-------------------|--------|---------|---------|---------|
| Number of players | 11 | 9 | 8 | 5 |

Calculate the median bowling speed.

17. The monthly income of 100 families are given as below :

| Income (in Rs) | Number of families |
|----------------|--------------------|
| 0-5000 | 8 |
| 5000-10000 | 26 |
| 10000-15000 | 41 |
| 15000-20000 | 16 |
| 20000-25000 | 3 |
| 25000-30000 | 3 |
| 30000-35000 | 2 |
| 35000-40000 | 1 |

Calculate the modal income.

18. The weight of coffee in 70 packets are shown in the following table :

| Weight (in g) | Number of packets |
|---------------|-------------------|
| 200-201 | 12 |
| 201-202 | 26 |
| 202-203 | 20 |
| 203-204 | 9 |
| 204-205 | 2 |
| 205-206 | 1 |

Determine the modal weight.

19. Two dice are thrown at the same time. Find the probability of getting

- (i) same number on both dice.
- (ii) different numbers on both dice.

20. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the dice is

- (i) 7?
- (ii) a prime number?
- (iii) 1?

- 21.** Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is
(i) 6 (ii) 12 (iii) 7
- 22.** Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9.
- 23.** Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3, respectively. They are thrown and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately.
- 24.** A coin is tossed two times. Find the probability of getting at most one head.
- 25.** A coin is tossed 3 times. List the possible outcomes. Find the probability of getting
(i) all heads (ii) at least 2 heads
- 26.** Two dice are thrown at the same time. Determine the probability that the difference of the numbers on the two dice is 2.
- 27.** A bag contains 10 red, 5 blue and 7 green balls. A ball is drawn at random. Find the probability of this ball being a
(i) red ball (ii) green ball (iii) not a blue ball
- 28.** The king, queen and jack of clubs are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is
(i) a heart (ii) a king
- 29.** Refer to Q.28. What is the probability that the card is
(i) a club (ii) 10 of hearts
- 30.** All the jacks, queens and kings are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1 similar value for other cards, find the probability that the card has a value
(i) 7 (ii) greater than 7 (iii) less than 7
- 31.** An integer is chosen between 0 and 100. What is the probability that it is
(i) divisible by 7? (ii) not divisible by 7?
- 32.** Cards with numbers 2 to 101 are placed in a box. A card is selected at random. Find the probability that the card has
(i) an even number (ii) a square number

- 33.** A letter of English alphabets is chosen at random. Determine the probability that the letter is a consonant.
- 34.** There are 1000 sealed envelopes in a box, 10 of them contain a cash prize of Rs 100 each, 100 of them contain a cash prize of Rs 50 each and 200 of them contain a cash prize of Rs 10 each and rest do not contain any cash prize. If they are well shuffled and an envelope is picked up out, what is the probability that it contains no cash prize?
- 35.** Box A contains 25 slips of which 19 are marked Re 1 and other are marked Rs 5 each. Box B contains 50 slips of which 45 are marked Re 1 each and others are marked Rs 13 each. Slips of both boxes are poured into a third box and resuffled. A slip is drawn at random. What is the probability that it is marked other than Re 1?
- 36.** A carton of 24 bulbs contain 6 defective bulbs. One bulb is drawn at random. What is the probability that the bulb is not defective? If the bulb selected is defective and it is not replaced and a second bulb is selected at random from the rest, what is the probability that the second bulb is defective?
- 37.** A child's game has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. One piece is lost at random. Find the probability that it is a
- (i) triangle (ii) square (iii) square of blue colour
(iv) triangle of red colour
- 38.** In a game, the entry fee is Rs 5. The game consists of a tossing a coin 3 times. If one or two heads show, Sweta gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. For tossing a coin three times, find the probability that she
- (i) loses the entry fee.
(ii) gets double entry fee.
(iii) just gets her entry fee.
- 39.** A die has its six faces marked 0, 1, 1, 1, 6, 6. Two such dice are thrown together and the total score is recorded.
- (i) How many different scores are possible?
(ii) What is the probability of getting a total of 7?
- 40.** A lot consists of 48 mobile phones of which 42 are good, 3 have only minor defects and 3 have major defects. Varnika will buy a phone if it is good but the trader will only buy a mobile if it has no major defect. One phone is selected at random from the lot. What is the probability that it is

- (i) acceptable to Varnika?
 (ii) acceptable to the trader?
41. A bag contains 24 balls of which x are red, $2x$ are white and $3x$ are blue. A ball is selected at random. What is the probability that it is
 (i) not red? (ii) white?
42. At a fete, cards bearing numbers 1 to 1000, one number on one card, are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square greater than 500, the player wins a prize. What is the probability that
 (i) the first player wins a prize?
 (ii) the second player wins a prize, if the first has won?

(E) Long Answer Questions

Sample Question 1: The following is the cumulative frequency distribution (of less than type) of 1000 persons each of age 20 years and above. Determine the mean age.

| | | | | | | |
|-----------------------------|-----|-----|-----|-----|-----|------|
| Age below (in years) | 30 | 40 | 50 | 60 | 70 | 80 |
| Number of persons | 100 | 220 | 350 | 750 | 950 | 1000 |

Solution : First, we make the frequency distribution of the given data and then proceed to calculate mean by computing class marks (x_i), u_i 's and $f_i u_i$'s as follows :

| Class | Frequency (f_i) | Class mark (x_i) | $u_i = \frac{x_i - 45}{10}$ | $f_i u_i$ |
|--------------|-------------------------------------|--------------------------------------|-----------------------------|----------------------|
| 20-30 | 100 | 25 | -2 | -200 |
| 30-40 | 120 | 35 | -1 | -120 |
| 40-50 | 130 | 45 | 0 | 0 |
| 50-60 | 400 | 55 | 1 | 400 |
| 60-70 | 200 | 65 | 2 | 400 |
| 70-80 | 50 | 75 | 3 | 150 |
| | $\sum f_i = 1000$ | | | $\sum f_i u_i = 630$ |

We have taken assumed mean (a) = 45. Here, h = class size = 10
 Using the formula

$$\begin{aligned} \text{Mean} = \bar{x} &= a + h \left(\frac{\sum f_i u_i}{\sum f_i} \right) \\ &= 45 + 10 \left(\frac{630}{1000} \right) \\ &= 45 + 6.3 = 51.3 \end{aligned}$$

Thus, the mean age is 51.3 years.

Sample Question 2: The mean of the following distribution is 18. The frequency f in the class interval 19-21 is missing. Determine f .

| | | | | | | | |
|-----------------------|-------|-------|-------|-------|-------|-------|-------|
| Class interval | 11-13 | 13-15 | 15-17 | 17-19 | 19-21 | 21-23 | 23-25 |
| Frequency | 3 | 6 | 9 | 13 | f | 5 | 4 |

Solution :

| Class interval | Mid-point (x_i) | Frequency (f_i) | $u_i = \frac{x_i - 18}{2}$ | $f_i u_i$ |
|-----------------------|-------------------------------------|-------------------------------------|----------------------------|------------------------|
| 11-13 | 12 | 3 | -3 | -9 |
| 13-15 | 14 | 6 | -2 | -12 |
| 15-17 | 16 | 9 | -1 | -9 |
| 17-19 | 18 | 13 | 0 | 0 |
| 19-21 | 20 | f | 1 | f |
| 21-23 | 22 | 5 | 2 | 10 |
| 23-25 | 24 | 4 | 3 | 12 |
| | | $\sum f_i = 40 + f$ | | $\sum f_i u_i = f - 8$ |

Let us take assumed mean (a) = 18. Here $h = 2$

$$\text{Mean} = \bar{x} = a + h \left(\frac{\sum f_i u_i}{\sum f_i} \right)$$

$$= 18 + 2 \left(\frac{f - 8}{40 + f} \right)$$

$$\bar{x} = 18 \text{ (Given)}$$

So,
$$18 = 18 + \frac{2(f - 8)}{40 + f}$$

or
$$f = 8$$

Hence, the frequency of the class interval 19-21 is 8.

Sample Question 3 : The median of the distribution given below is 14.4. Find the values of x and y , if the total frequency is 20.

| | | | | | |
|-----------------------|-----|------|-------|-------|-------|
| Class interval | 0-6 | 6-12 | 12-18 | 18-24 | 24-30 |
| Frequency | 4 | x | 5 | y | 1 |

Solution :

| Class interval | Frequency | Cumulative frequency |
|-----------------------|------------------|-----------------------------|
| 0-6 | 4 | 4 |
| 6-12 | x | $4 + x$ |
| 12-18 | 5 | $9 + x$ |
| 18-24 | y | $9 + x + y$ |
| 24-30 | 1 | $10 + x + y$ |

It is given that $n = 20$.

So, $10 + x + y = 20$, i.e., $x + y = 10$ (1)

It is also given that median = 14.4

which lies in the class interval 12-18.

So, $l = 12$, $f = 5$, $cf = 4 + x$, $h = 6$

Using the formula

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) h$$

$$\text{we get, } 14.4 = 12 + \left(\frac{10 - (4 + x)}{5} \right) 6$$

$$\text{or } 14.4 = 12 + \left(\frac{6 - x}{5} \right) 6$$

$$\text{or } x = 4 \quad (2)$$

From (1) and (2), $y = 6$.

EXERCISE 13.4

1. Find the mean marks of students for the following distribution :

| Marks | Number of students |
|---------------|--------------------|
| 0 and above | 80 |
| 10 and above | 77 |
| 20 and above | 72 |
| 30 and above | 65 |
| 40 and above | 55 |
| 50 and above | 43 |
| 60 and above | 28 |
| 70 and above | 16 |
| 80 and above | 10 |
| 90 and above | 8 |
| 100 and above | 0 |

2. Determine the mean of the following distribution :

| Marks | Number of students |
|-----------|--------------------|
| Below 10 | 5 |
| Below 20 | 9 |
| Below 30 | 17 |
| Below 40 | 29 |
| Below 50 | 45 |
| Below 60 | 60 |
| Below 70 | 70 |
| Below 80 | 78 |
| Below 90 | 83 |
| Below 100 | 85 |

3. Find the mean age of 100 residents of a town from the following data :

| Age equal and above (in years) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 |
|--------------------------------|-----|----|----|----|----|----|----|----|
| Number of Persons | 100 | 90 | 75 | 50 | 25 | 15 | 5 | 0 |

4. The weights of tea in 70 packets are shown in the following table :

| Weight (in gram) | Number of packets |
|------------------|-------------------|
| 200-201 | 13 |
| 201-202 | 27 |
| 202-203 | 18 |
| 203-204 | 10 |
| 204-205 | 1 |
| 205-206 | 1 |

Find the mean weight of packets.

5. Refer to Q.4 above.

Draw the less than type ogive for this data and use it to find the median weight.

6. Refer to Q.4 above.

Draw the less than type and more than type ogives for the data and use them to find the median weight.

7. The table below shows the salaries of 280 persons.

| Salary (in thousand (Rs)) | Number of persons |
|------------------------------|-------------------|
| 5-10 | 49 |
| 10-15 | 133 |
| 15-20 | 63 |
| 20-25 | 15 |
| 25-30 | 6 |
| 30-35 | 7 |
| 35-40 | 4 |
| 40-45 | 2 |
| 45-50 | 1 |

Calculate the median and mode of the data.

8. The mean of the following frequency distribution is 50, but the frequencies f_1 and f_2 in classes 20-40 and 60-80, respectively are not known. Find these frequencies, if the sum of all the frequencies is 120.

| Class | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 |
|-----------|------|-------|-------|-------|--------|
| Frequency | 17 | f_1 | 32 | f_2 | 19 |

9. The median of the following data is 50. Find the values of p and q , if the sum of all the frequencies is 90.

| Marks | Frequency |
|-------|-----------|
| 20-30 | p |
| 30-40 | 15 |
| 40-50 | 25 |
| 50-60 | 20 |
| 60-70 | q |
| 70-80 | 8 |
| 80-90 | 10 |

10. The distribution of heights (in cm) of 96 children is given below :

| Height (in cm) | Number of children |
|----------------|--------------------|
| 124-128 | 5 |
| 128-132 | 8 |
| 132-136 | 17 |
| 136-140 | 24 |
| 140-144 | 16 |
| 144-148 | 12 |
| 148-152 | 6 |
| 152-156 | 4 |
| 156-160 | 3 |
| 160-164 | 1 |

Draw a less than type cumulative frequency curve for this data and use it to compute median height of the children.

11. Size of agricultural holdings in a survey of 200 families is given in the following table:

| Size of agricultural holdings (in ha) | Number of families |
|---------------------------------------|--------------------|
| 0-5 | 10 |
| 5-10 | 15 |
| 10-15 | 30 |
| 15-20 | 80 |
| 20-25 | 40 |
| 25-30 | 20 |
| 30-35 | 5 |

Compute median and mode size of the holdings.

12. The annual rainfall record of a city for 66 days is given in the following table.

| Rainfall (in cm) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
|------------------|------|-------|-------|-------|-------|-------|
| Number of days | 22 | 10 | 8 | 15 | 5 | 6 |

Calculate the median rainfall using ogives (of more than type and of less than type)

13. The following is the frequency distribution of duration for 100 calls made on a mobile phone :

| Duration (in seconds) | Number of calls |
|-----------------------|-----------------|
| 95-125 | 14 |
| 125-155 | 22 |
| 155-185 | 28 |
| 185-215 | 21 |
| 215-245 | 15 |

Calculate the average duration (in sec) of a call and also find the median from a cumulative frequency curve.

14. 50 students enter for a school *javelin* throw competition. The distance (in metres) thrown are recorded below :

| Distance (in m) | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 |
|--------------------|------|-------|-------|-------|--------|
| Number of students | 6 | 11 | 17 | 12 | 4 |

- Construct a cumulative frequency table.
- Draw a cumulative frequency curve (less than type) and calculate the median distance thrown by using this curve.
- Calculate the median distance by using the formula for median.
- Are the median distance calculated in (ii) and (iii) same?