6
Chapter

Similar Triangles

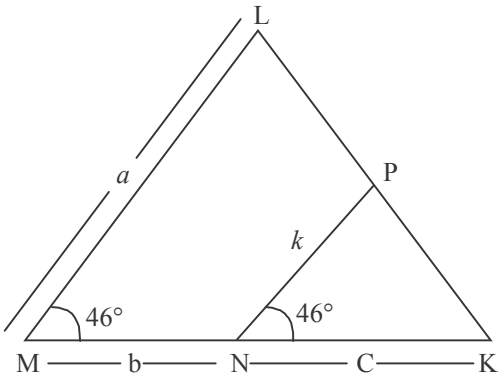
Key Points

1. **Similar Triangles :** Two triangles are said to be similar if their corresponding angles are equal and their corresponding sides are proportional.
2. **Criteria for Similarity :**
   in ΔABC and ΔDEF
   (*i*) **AAA Similarity :** $\Delta ABC \sim \Delta DEF$ when $\angle A\ \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$
   (*ii*) SAS similarity :
   $\Delta ABC \sim \Delta DEF$ when $\frac{AB}{DE} = \frac{BC}{EF}$ and $\angle B = \angle E$
   (iii) SSS Similarty: $\Delta ABC \sim \Delta DEF$, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$
3. **The proof of the following theorems can be asked in the exmination :**
   (*i*) Basic Proportionality Theoren: If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the same ratio.
   (*ii*) The ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
   (*iii*) **Pythagoras Theorm:** In a right triangles the square of the hypotenuse is equal to the sum of the squares of the other two sides.
   (*iv*) Converse of pythagoras thearem— In a triangle, if the square of one side is equal to the sum of squares of other two sides then the angle oppo site to the first side is a right angle

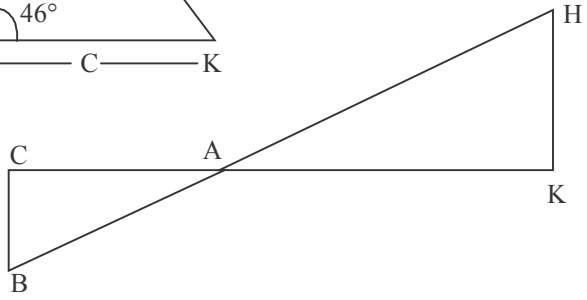
1. Is the triangle with sides 12cm, and 18 cm a right triangle? Give reason.

2. If $\Delta ABC \sim \Delta QRP$, $\frac{\text{Area } (\Delta ABC)}{\text{Area } (\Delta PQR)} = \frac{9}{4}$, $AB = 18cm$, $BC = 15cm$, then find the length of PR.

3. In the fig., LM = LN = 46°, Express x in terms of a, b and c.



4. In fig. ΔAHK ~ ΔABC.
If AK = 10*cm*, BC = 3.5*cm* and HK = 7*cm*, find AC.



5. It is given that ΔDEF ~ ΔRPQ. Is it trne to say that ∠D = ∠R and ∠F = ∠P?

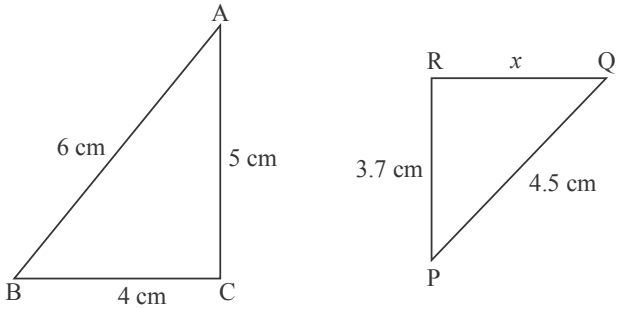
6. If the corresponding Medians of two similar triangles are in the ratio 5 : 7, Then Find the ratio of their sides.

7. A right angled triangle has its area numerically equal to its perimeter. The length of each side is an even number and the hypotenuse is 10cm. What is the perimeter of the triangle?

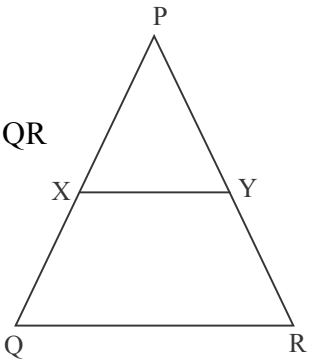
8. An aeroplane leaves an airport and flies due west at a speed of 2100 km/hr. At the same time, another aeroplane leaves the same place at airport and flies due south at a speed of 2000 km/hr. How far apart will be the two planes after 1 hour?

9. The areas of two similar ΔABC and ΔDEF are 225 cm^2 and 81 cm^2 respectively. If the longest side of the larger triangle ΔABC be 30 cm, find the langest side of the smaller triangle DEF.

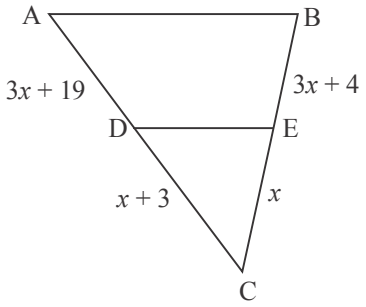
10. In the figure, if ΔABC ~ ΔPQR, find the value of x?



11. In the figure, XY || QR and $\frac{PX}{XQ}=\frac{PY}{YR}=\frac{1}{2}$, find XY : QR



12. In figure, find the value of x which will make DE || AB?



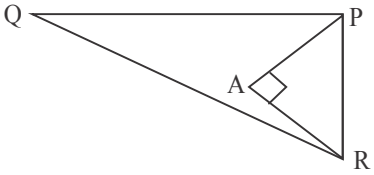
13. If ΔABC ~ ΔDEF, BC = 3EF and ar (ΔABC) = 11 7cm^2 find area (ΔDEF).

14. If ΔABC and ΔDEF are similar triangles such that ∠A = 45° and ∠F = 56°, then find ∠C.

15. If the ratio of the corresponding sides of two similar triangles is 2 : 3, then find the ratio of their corresponding attitudes.

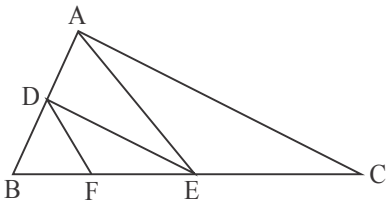
SHORT ANSWER TYPE (I) QUESTIONS

16. In the given fig. PQ = 24cm , QR = 26cm, ∠PAR = 90°, PA = 6cm and AR = 8cm, find ∠QPR.



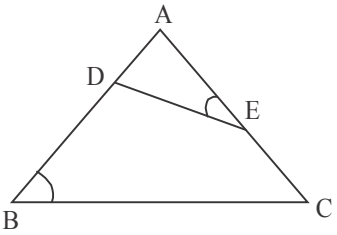
17. In the given fig., DE || AC and DF || AE. Prove that

$\frac{FE}{BF}=\frac{EC}{BE}$

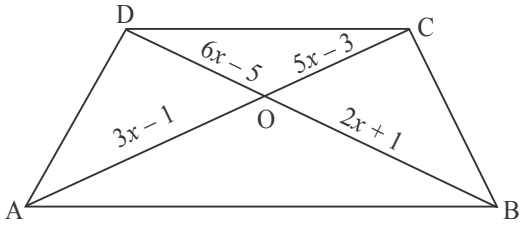


18. In ΔABC, $AD \perp BC$ Such that $AD^2 = BD \times CD$. Prove that ΔABC is right angled at A.

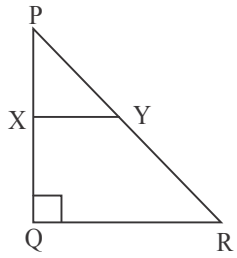
19. In the given fig, D and E are points on sides AB and CA of ΔABC such that $\angle B = \angle AED$. Show that $\Delta ABC \sim \Delta AED$.



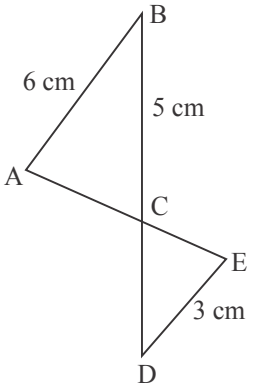
20. In the given fig., AB || DC and diagonals AC and BD intersects at O. If OA = $3x$ –1 and OB = $2x + 1$, OC = $5x - 3$ and OD = $6x - 5$, find x.



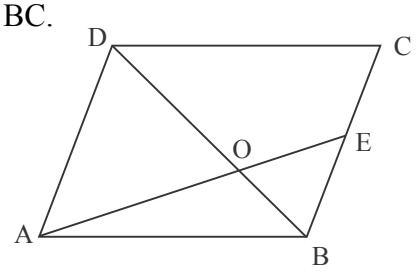
21. In the fig, PQR is a triangle, right angled at Q. If XY || QR, PQ = 6cm, PY = 4cm & PX : XQ = 1 : 2 Calculate the lengths of PR and QR.



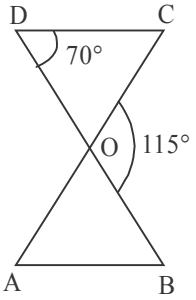
22. In the figure, AB || DE. Find the length of CD.



23. In the figure, ABCD is a parallelogram. AE divides the line segment BD in the ratio 1 : 2. If BE = 1.5cm find BC.



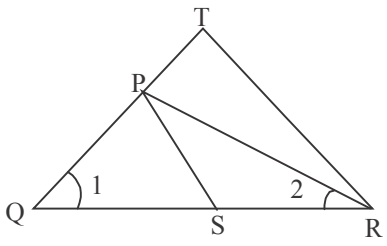
24. In the given figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 115°$ and $\angle CDO = 70$ find, (i) $\angle DOC$, (ii) $\angle DCO$, (iii) $\angle OAB$, (iv) $\angle OBA$.



25. Perimeter of two equilateral triangles ABC and PQR are 144m and 96m, find ar (ΔABC) : ar (ΔPQR)

SHORT ANSWER TYPE (II) QUESTION

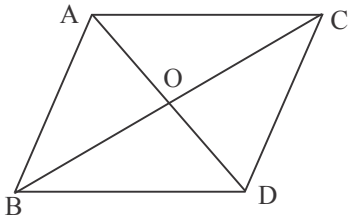
26. In the figure, $\frac{QR}{QS}=\frac{QT}{PR}$ and $\angle 1 = \angle 2$ them prove that $\Delta PQS \sim \Delta TQR$



27. In equilateral ΔABC, $AD \perp BC$. Prove that $3\ BC^2 = 4AD^2$.

28. In ΔABC, $\angle ACB = 90°$, also $CD \perp AB$, Prove that $\frac{BC^2}{AC^2}=\frac{BD}{AD}$.

29. In the adjoining figure ΔABC & ΔDBC are on the same base BC. AD & BC intersect at O. Prove that $\frac{\text{area }(\Delta ABC)}{\text{area }(\square DBC)} = \frac{AO}{DO}$



30. In ΔABC, If AD is the median, Show that $AB^2 + AC^2 = 2(AD^2 + BD^2)$

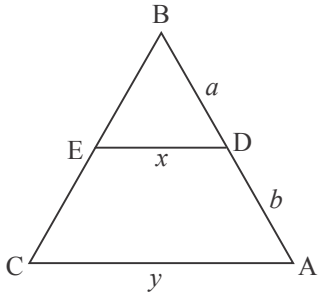
31. In ΔABC, ∠C is a right angle. Points P & Q lies on the sides CA & CB respectively Prove that $AQ^2 + BP^2 = AB^2 + PQ^2$

32. If AD and PS are medians of ΔABC and ΔPQR respectively where ΔABC ~ ΔPQR, Prove that $\frac{AB}{PQ} = \frac{AD}{PS}$.

33. In an equilateral ΔABC, AD ⊥ BC, Prove that $3AB^2 = 4AD^2$

34. In the given fig, DE || AC. which of the following is correct?

$x = \frac{a+b}{ay}$ or $x = \frac{ay}{a+b}$

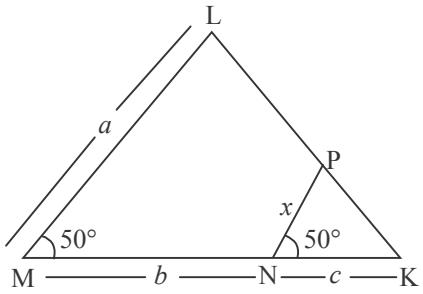


35. Prove that the sum of the square of the sides of a rhombus is equal to the sum of the squares of its diagonals;

36. A street light bulb is fixed on a pole 6*m* above the level of the street. If a woman of height 1.5*m* casts a shadow of 3*m*, find how for she is away from the base of the pole.

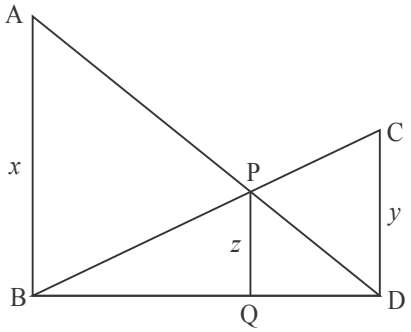
37. Two poles of height *a* metres and *b* metres are p metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is gives by $\frac{ab}{a+b}$ metres.

38. In the given fig., find the value of x in terms of a, b and c

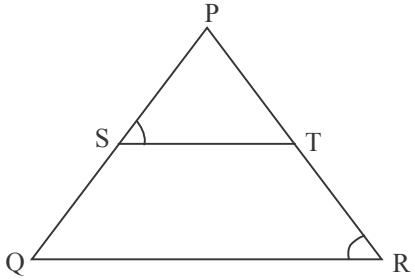


39. In fig., AB || PQ || CD, AB = x units. CD = y units and PQ = z units. Prove that

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$$

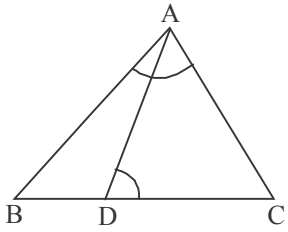


40. In the given fig., $\frac{PS}{SQ}=\frac{PT}{TR}$ and ∠PST = ∠PRQ. Prove that PQR is an isosceles Δ.

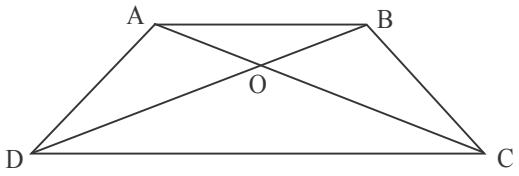


41. In the figure, D is a point on the side BC of ΔABC such that ∠ADC = ∠BAC

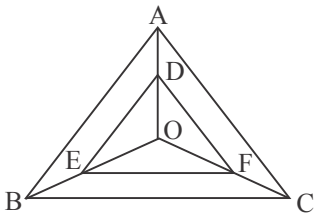
Prove that $\frac{CA}{CD}=\frac{CB}{CA}$



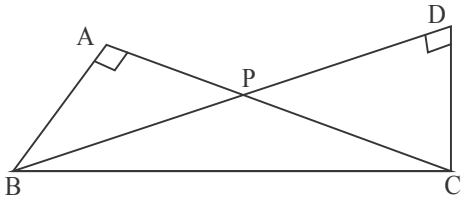
42. In the figure, ABCD is a trapezium in which AB || DC, the diagonals AC & BD intersect at O. Prove that $\frac{AO}{OC}=\frac{BO}{DO}$



43. In the figure, a point O inside ΔABC is joined to its vertices. From a point D on AO, DE is drawn parallel to AB & from E, EF is drawn parallel to BC. Prove that DF || AC.



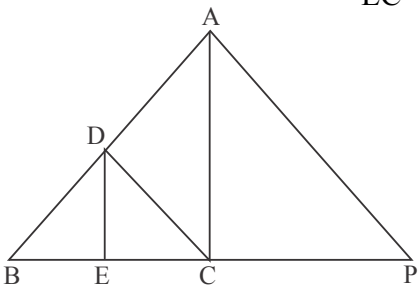
44. Two triangles BAC and BDC, right angled at A and D respectively, are drawn on the same base BC and on the same side of BC. If AC and DB intersect at P, Prove that AP × PC = DP × PB



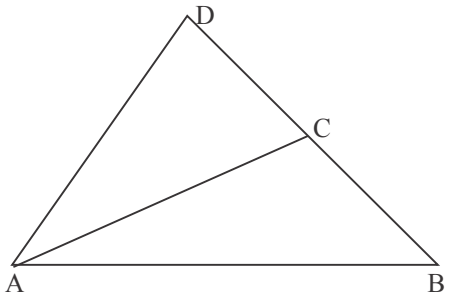
45. Hypotenuse of a right triangle is 25 cm and out of the remaining two sides, one is larger than the other by 5 cm, find the lengths of the other two sides.

LONG ANSWER TYPE QUESTIONS

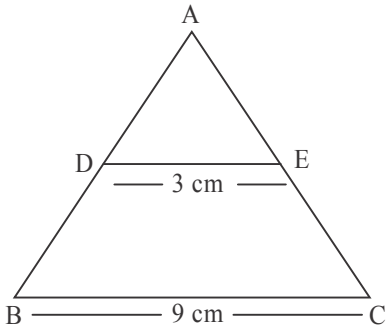
46. In the following figur, DE || AC and $\frac{BE}{EC}=\frac{BC}{CP}$. Prove that DC || AP.



47. In a quadrilateral ABCD, $\angle B = 90°$, $AD^2 = AB^2 + BC^2 + CD^2$. Prove that $\angle ACD = 90°$



48. In figure, DE || BC, DE = 3cm, BC = 9cm and ar(ΔADE) = 30cm^2. Find ar (trap. BCED).



49. State and prove Pythagoras theorem.

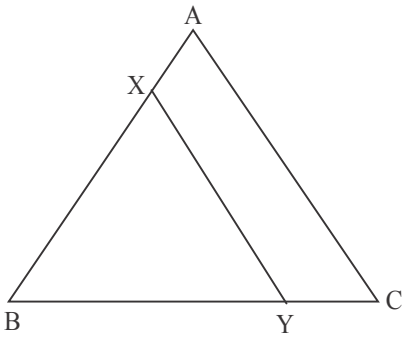
50. In an equilateral ΔABC, D is *a* point on side BC such that $BD = \frac{1}{3}$ BC. Prove that $9AD^2 = 7AB^2$.

51. IN ΔPQR, $PD \perp QR$ such that D lies on QR. If PQ = *a*, PR = *b*, QD = *c* and DR = d and *a*, *b*, *c*, *d* are positive units. Prove that $(a + b)(a - b) = (c + d)(c - d)$.

52. In a trapezium ABCD, AB || DC and DC = 2AB. If EF is drawn parallel to AB cuts AD in F and BC in E such that $\frac{BE}{BC} = \frac{3}{4}$. Diagonals DB intersects EF at G Prove that 7 EF = 10AB.

53. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

54. In the given figure, the line segment XY is Parallel to AC of ΔABC and it divides the triangle into two parts of equal areas. Prove that $\frac{AX}{AB}=\frac{\sqrt{2}-1}{\sqrt{2}}$



55. Through the vertex D of a parallelogram ABCD, a line is drawn to intersect the sides BA and BC produced at E and F respectively. Prove that

$$\frac{DA}{AE}=\frac{FB}{BE}=\frac{FC}{CD}$$

56. Prove dthat if in a triangle, the square on one side is equal to the sum of the squares on the other two sides, then the angle opposite to the first side is a right angle.

ANSWERS

1. No

2. 10cm

3. $x = \frac{ac}{b+c}$

4. 5cm

5. $\angle D = \angle R$ true, $\angle F = \angle P$ false

6. 5 : 7

7. 24cm

8. 2900km

9. 18cm

10. x = 3

11. 1 : 3

12. x = 2

13. 13cm^2

14. 56°

15. 2 : 3

16. 90°

20. $x = 2$

21. PR = 12cm, QR $6\sqrt{3}$cm

22. 2.5 cm

23. 3cm

24. 65°, 45°, 45°, 70°

25. 9 : 4

34. $x = \frac{ay}{a+b}$

36. 9m

38. $x = \frac{ac}{b+c}$

45. 15cm, 20cm

48. 240 cm^2

Practice-Test

Similar Triangles

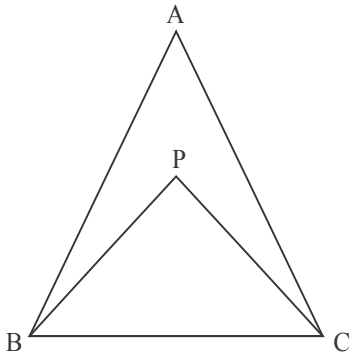
MM: 20 ***Duration : 50 Minutes***

1. The lengths of the diagonals of rhombus are 16cm and 12cm. find the side of the rhombus. 1

2. In an equilateral ΔABC , $AD \perp BC$ and $\frac{AD^2}{BC^2} = x$ find the volue of x. 1

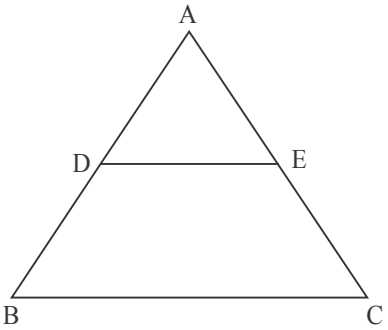
3. In ΔABC, if $DE \parallel BC$, $AD = x + 1$, $DB = x - 1$, $AE = x + 3$ and $EC = x$, then find the value of x. 2

4. In the given figure, can triangle ABC be similar to ΔPBC? If yes, give reasons. 2



5. PQR is a right angled triangle, having $\angle Q = 90°$, If QS = SR, Show that $PR^2 = 4PS^2 - 3PQ^2$. 3

6. In figure, DE || BC and AD : DB = 5 : 4, find $\frac{\text{Area } (\square DFE)}{\text{Area } (\square CFB)}$ 3



7. State and prove pythagoras theorem. 4

8. In as equilateral ΔLMN, the side MN is trisectedat O. prove that $\frac{LO^2}{LM^2} = \frac{7}{9}$. 4

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