

Polynomials

Key Points

- Polynomial :** If x is a variable, n is a natural number and $a_0, a_1, a_2, a_3, \dots, a_n$ are real numbers, then $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, ($a_n \neq 0$) is called a polynomial in x .
- Polynomials of degree 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.
- A quadratic polynomial is an algebraic expression of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.
- Zeroes of a polynomial $p(x)$ are precisely the x – coordinates of the points where the graph of $y = p(x)$ intersects the x –axis, *i.e.*, $x = a$ is a zero of polynomial $p(x)$ if $p(a) = 0$
- A polynomial can have at most the same number of zeroes as the degree of the polynomial.
- If one zero of a quadratic polynomial $p(x)$ is negative of the other, then co-efficient of $x = 0$
 - If zeroes of a quadratic polynomial $p(x)$ are reciprocal of each other, then co-efficient of $x^2 =$ constant term.
- Relationship between zeroes and coefficients of a polynomial
If α And β Are zeroes of $p(x) ax^2 + bx + c$ ($a \neq 0$), then
Sum of zeroes $= \alpha + \beta = -\frac{b}{a}$
Product of zeroes $= \alpha\beta = \frac{c}{a}$
- If α, β are zeroes of a quadratic polynomial $p(x)$, then
 $p(x) = k [x^2 - (\text{sum of zeroes}) x + \text{product of zeroes}]$
 $\Rightarrow p(x) = k [x^2 - (\alpha + \beta)x + \alpha\beta]$; where k is any non-zero real number.
- Graph of linear polynomial $p(x) = ax + b$ is a straight line.
- Division Algorithm states that given any polynomials $p(x)$ and $g(x)$, there exist polynomial $q(x)$ and $r(x)$ such that:

$$p(x) = g(x). q(x) + r(x) ; g(x) \neq 0,$$

[where either $r(x) = 0$ or degree $r(x) <$ degree $g(x)$]