Mathematics - Class XI

Chapter - 1 (SETS)

Introduction

In our daily life, while performing our regular work, we often come across a variety of things that occur in groups e.g. (i) Army of soldiers (ii) Team of cricket players (iii) Group of pretty girls (iv) Pack of playing cards (v) Bunch of beautiful flowers.

The words used above like Army, Team, Group, Bunch, Pack etc. convey the idea of certain collections.

Well defined collection of objects:

If any given collection of objects is in such a way that it is possible to tell, without any doubt whether a given object belong to this collection or not, then such a collection of objects is called a well defined collection of objects.

Some more examples of well defined collections:

(i) Vowels of english alphabet

(ii) Odd natural number less than 10.

(iii) The rivers of India

(iv) Prime factor of 210, namely 2, 3, 5 and 7

(v) The solution of equation : $x^2 - 5x + 6 = 0$ viz 2 and 3.

SET : The theory of sets was developed by German Mathematician Cantor (1845 - 1918 AD).

Definition : A well defined collection of distinct objects is called a set.

Here the following points are noted :

(i). Objects, elements and members of a set are synonymous words.
(ii) Sets are usually denoted by the capital letters A, B, C, X, Y, Z etc.

(iii) The elements of sets are represented by small letters a, b, c, x, y, z etc.

If a is an element of a set A, then we say that a belongs to A. The word 'belongs to' is denoted by the Greek symbol $\in$ (epsilon). Thus, in notation form, a belongs to A is written as $a \in A$ and b does not belong to A is written as $b \notin A$.

Thus, in the set V of vowels in the English alphabet, $a \in V$ but $b \notin V$. In the set P of prime factors of 30, $3 \in P$ but $15 \notin P$.

Similarly, (i) $5 \in N$, N being a set of natural numbers, $0 \notin N$.

(ii) $25 \in P$, P being a set of perfect square numbers.

(iii) $2 \in Z$, Z being set of integers, $0.5 \notin N$.

Representation of Sets.

Sets are generally represented by the following two ways:
(1) Roster or Tabular form (2) Rule Method or Set builder form.

(1) Roster or Tabular form:

In this form, all the elements of a set are listed, the elements are being separated by commas are are enclosed with brackets { }.

e.g. (i) The set of all natural numbers less than 5 a $\{1,2,3,4\}$
(ii) The set of all factors of 50 is $\{1,2,5,10,25,50\}$
(iii) The set of all distinct Letters in MATHEMATICS is $\{M, A, T, H, E, I, C, S\}$.
(iv) The set of prime natural numbers is $\{2, 3, 5, 7, \ldots\}$

Note: (1) In a set notation, order is not important. Hence the set of all factors of 50 can be $\{1, 5, 10, 2, 50, 25\}$.
(2) The elements of set are generally not repeated.
(3) The dot(s) $(\ldots)$ reveals that the list is endless (continued)

(2) Rule Method or Set builder form:

In this form, all the elements of a set possess a single common property $p(x)$, which is not possessed by any other element outside set. We describe by $\{x : p(x) \text{ holds}\}$

If we denote the set of vowels by V, then we write
$V = \{x : x$ is a vowel in the English alphabet$\}$

It is read as "The set of all x such that x is a vowel of the English alphabet".

More examples, (i) Set of all natural numbers less than 5
$A = \{x : x \in N,\ x < 5\}$

(ii) Set of all integers $Z = \{x : x \in Z\}$
(iii) Set of months having 31 days
$A = \{x : x$ is the month of a year having 31 days$\}$

Note: any other symbol like, y, z etc could be used in place of x.

Examples - I

(1) Write the solution set of the equation $x^2+x-2=0$ in roster form.

(2) Write the set $\{x : x$ is a positive integer and $x^2<40\}$ in the roster form.

(3) Write the set $A=\{1, 4, 9, 16, 25, \ldots\}$ in set builder form.

(4) Write the set $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\}$ in the set builder form.

(5) Match each of the following:

(i) $\{P, R, I, N, C, A, L\}$	(a) $\{x : x$ is a positive integer and is a divisor of $18\}$
(ii) $\{0\}$	(b) $\{x : x$ is an integer and $x^2-9=0\}$
(iii) $\{1, 2, 3, 6, 9, 18\}$	(c) $\{x : x$ is an integer and $x+1=1\}$
(iv) $\{3, -3\}$	(d) $\{x : x$ is a letter of the word PRINCIPAL$\}$

Statement	Roster form	Set Builder form
(1) The set of currencies used in USA, UK, Japan, Germany and Russia	{Dollor, Pound, Yen, Euro, Rouble}	{x : x is the currency used in USA, UK, Japan Germany and Russia.
(2) The set of all distinct letters used in the word Students	{S, T, U, D, E, N, T}	{x : x is the distinct letters used in the word STUDENT}
(3) The set of all natural numbers between 11 and 15	$\{12, 13, 14\}$	$\{x : x \in N, 11<x<15\}$
(4) The set of all the states of India beginning with letter A.	{Andhra Pradesh, Arunachal Pradesh, Assam}	{x : x is the state of India beginning with the letter A}
(5) The of Capitals of Kerala, Karnatak, Tamil Nadu, Andha Pradesh, Gujrat.	{Thiruvanathpuram, Bengaluru, Chennai, Hydrabad and Gandhi Nagar}	{x : x is the capitals of Kerala, Karnataka, Tamilnadu, Andhra Pradesh and Gujrat.

(6) Which of the following collections are sets.?

(i) The collection of all months of a year, beginning with letter J.

(ii) The collection of most talented writers of India

(iii) The collection of all natural numbers less than 100

(iv) A collection of most dangerous animals of the world.

7) Let $A = \{1, 2, 3, 4, 5, 6\}$. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces.

(i) 5 ...... A, (ii) 8 ..... A (iii) 0 ..... A (iv) 4 ..... A

8) Write the set of all consonants in the English alphabet which precede k.

9) Write the set of all Indian Miss World since 1995.

10) Write the following sets in Roster form:

(i) The set of all natural numbers 'x' such that $4x+9 < 50$

(ii) The set of all positive integers 'x' such that $|x-3| < 8$

11) Describe the following sets in Set builder form:

(i) $B = \{-1, 1\}$ (ii) $C = \{14, 21, 28, 35, 42, ..., 98\}$

(iii) $A = \{53, 59, 61, 67, 71, 73, 79, 83, 89, 97\}$

12) Write the set of all even integers whose cube is odd.

13) Write the set $\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}\}$ in the set builder form.

14) Write the set $A = \{x : x$ is an integer and $x^2 < 20\}$ in the Roster form.

15) Write the set $A = \{x : x$ is a planet$\}$ in the Roster form.

Solution

(1) The given equation $x^2+x-2=0$ can be factorised as:

$$(x-1)(x+2)=0 \quad \text{i.e. } x=1,-2.$$

$\therefore$ Solution in roster form as $\{1,-2\}$

(2) The required numbers are 1, 2, 3, 4, 5, 6.
The given set in roaster form is $\{1,2,3,4,5,6\}$

(3) Let $A = \{x : x$ is a square of natural number$\}$

We can also write $A = \{x : x = n^2$, where $n \in N\}$

(4) Let $A = \{x : x = \frac{n}{n+1}$, where n is a natural number$\}$

(5) (i) — (d), (ii) — (c) (iii) — (a) (iv) (b)

(6) (i) There are 3 months ie January, June and July beginning with letter J. Therefore, it is a well defined collection of months and hence, it is a set.

(ii) The concept of most talented writers of India is vague, since there is no rule given for deciding whether a particular writer is talented or not. Hence, the given collection is not a set.

(iii) The collection of all natural numbers less than 100 i.e. $\{1, 2, \ldots 99\}$ is well defined. Hence, the given collection is set

(iv) The concept of most dangerous animals of the world is vague, as there is no rule given for deciding.

(7) (i) $5 \in A$ (ii) $8 \notin A$ (iii) $0 \notin A$ (iv) $4 \in A$

(8) The consonants which proceeds k are: b, c, d, f, g, h, j;

Thus $A = \{b, c, d, f, g, h, j\}$

(9) A = { Aishwarya Rai, Yukta Mukhi, Lara Datta, Priyanka Chopra }.

(10) (i) $4x + 9 < 50 \Rightarrow 4x < 41 \Rightarrow x < \frac{41}{4} \Rightarrow x < 10.25$

Since x is natural number

$\therefore$ Required set $= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(ii) $|x-3| < 8$ case (i) $x - 3 < 8$, $x < 11$

$\therefore x = 3, 4, 5, 6, 7, 8, 9, 10, 11$

case (ii) $-(x-3) < 8 \Rightarrow x > -5$

$x = -4, -3, -2, -1, 0, 1, 2$

But we have to consider only positive value of x.

Thus Given set $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(11) (i) Given set $B = \{-1, 1\}$ $|x| = 1$

$\therefore$ Required set $= \{x : |x| = 1 \text{ and } x \in Z\}$

(ii) $C = \{14, 21, 28, 35, 42, \dots, 98\}$

We observe that numbers are multiple of 7 less than 100.

Given set = { x : x is a natural number multiple of 7 and $7 < x < 100$ }

(iii) $A = \{53, 59, 61, 67, 71, 73, 79, 83, 89, 97\}$

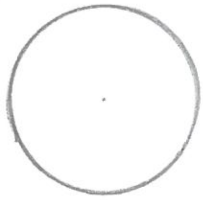
Given numbers are prime numbers between 50 & 100.

Given set = { x : x is a prime number and $50 < x < 100$ }

Types of Sets

(1) **Empty Set:** A set which does not contain any element is called an empty set or null or void set.

It is denoted by ϕ or $\{\ \}$



Empty Set.

For example: Let us consider the set
$A = \{x : x$ is a man living on the moon$\}$

We know that there is no man living on the moon. Therefore, the set A contain no element and hence, it is an empty set.

(i) Let $A = \{x : 1 < x < 2$, x is a natural number$\}$.
Then A is the empty set, because there no natural number between 1 and 2.

(ii) $B = \{x : x^2 - 2 = 0$ and x is rational number$\}$
Then B is the empty set because the $x^2 - 2 = 0$ is not satisfied by any rational value of x.

(2) Singleton Set:

A set, consisting of a single element is called a singleton set. The sets $\{0\}$, $\{5\}$, $\{-7\}$ are singleton sets.

$\{x : x + 6 = 0,\ x \in \mathbb{Z}\}$ is a singleton set, because this set contain only one integer namely, -6.

Let $B = \{x : x$ is a even prime number$\}$
Here B is a singleton set because there is only one prime number which is even i.e. 2

Let $A = \{x : x$ is neither prime nor composite$\}$
It is a singleton set containing one element i.e. 1

(3) Finite Set :

A set, which is empty or consists of a definite number of elements is called a finite set. e.g.

(i) The set $\{3, 4, 5, 6\}$ is a finite set because, it contains a definite number of elements i.e. only 4 elements.

(ii) The set of days in week is a finite set, because it contain a definite number of elements i.e. 7 days.

(iii) The set of solutions of $x^2 = 36$ is a finite set. Because it contains a definite number of elements i.e. only 6 and -6.

(iv) An empty set, which does not contain any element (no element) is also a finite set.

(4) Infinite Set :

A set, whose elements cannot be listed by the natural numbers 1, 2, 3, ... n, for any natural number n is called an infinite set. e.g.

(i) The set of squares of natural number is an infinite set, because such natural numbers are infinite.

(ii) The set of concentric circles is an infinite set.

(iii) The set of all points in a plane is an infinite set.

(iv) Set of natural numbers, rational numbers, real numbers, integers etc. are all examples of infinite sets.

Equal and Equivalent Sets

Equal Sets: Two sets A and B are said to be equal if every element of A is also an element of B and every element of B is also an element of A., two sets A and B are said to be equal if they have exactly the same elements and write $A = B$.

let $A = \{1, 2, 4\}$ and $B = \{4, 2, 1\}$ then $A = B$.

Equivalent Sets: Two finite sets A and B are said to equivalent if they have the same numbers of elements i.e. if $n(A) = n(B)$, then sets A and B are equivalent.

let $A = \{a, e, i, o, u\}$ $B = \{1, 2, 3, 4, 5\}$
$n(A) = 5$, $n(B) = 5$ $\therefore n(A) = n(B)$.

Note: Equal sets are equivalent, but equivalent sets are not always equal.

Subsets

1) If two sets A and B are such that every element of set A is in set B, we say that A is a subset of B and write $A \subseteq B$. "$\subseteq$" stands for "is a subset of".

let $A = \{a, e, i, o, u\}$ $B = \{a, b, c, d, e, \ldots, x, y, z\}$
Every element of set A is in set B.

Examples: $\{1\}$, $\{1, 2\}$, $\{1, 3\}$ are subsets of $\{1, 2, 3\}$

If A is not a subset of a set B, then we write $A \not\subseteq B$.

(2) $A \subseteq A$ for every set A, i.e. every set is a subset of itself.

(3) $\phi \subseteq A$ i.e. empty set is a subset of any other set.

(4) If $A \subseteq B$ and $B \subseteq A$, then $A = B$.

(5) If A is a subset of B, then B is called the super set of A.

(6) If A has n elements, then A has 2^n subsets.

(7) Proper Subset : A is called a proper subset of B if A is a subset of B and $A \neq B$ and this relationship is denoted by the symbol $A \subset B$ and is read as 'A is a proper subset of B'.

Power Set

A set formed by all the subsets of a Set A as its elements are called the power set of A and is denoted by $P(A)$. Let $A = \{a, b\}$

Subsets of A are $\phi, \{a\}, \{b\}$ and $\{a, b\}$

then power set of A, $P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

Comparability of Sets

Let A and B be two sets. If $A \subseteq B$ or $B \subseteq A$, then the sets A and B are said to be comparable.

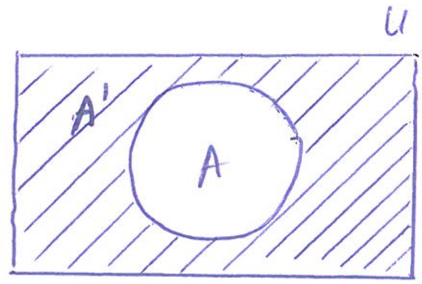
$A = \{1, 5\}$ $B = \{1, 3, 5\}$ are comparables as $A \subseteq B$.

but $A = \{1, 2, 5\}$ $B = \{1, 5, 7\}$ are incomparable as $2 \in A$ but $2 \notin B$ and $7 \in B$ but $7 \notin A$.

Universal Sets : If all the sets under consideration are subsets of a fixed set U, then this fixed set U is called the universal set or universe of discourse.

Complement of Set

If U is the universal set and A is a subset of U, then the complement of A is the set which contains those elements of U, which are not contained in A and is denoted by A' or A^c.



If $U = \{1, 2, 3, 4, \ldots\}$ $A = \{2, 4, 6, 8, \ldots\}$
then $A' = \{1, 3, 5, 7, \ldots\}$

Symbolically, $A' = U - A = \{x : x \in U \text{ and } x \notin A\}$
$= \{x : x \in U : x \notin A\}$

We can also define A' as follows:
A' is the set of all those elements which do not belong to A.
This shows that any $x \in A' \iff x \notin A$.

__Remarks:__ (i) $U' = \phi$ (ii) $\phi' = U$ (iii) $A \cup A' = U$
(iv) $A \cap A' = \phi$ (v) $(A')' = A$. [$\cup$ – union of sets].

__Properties of Complement Sets:__

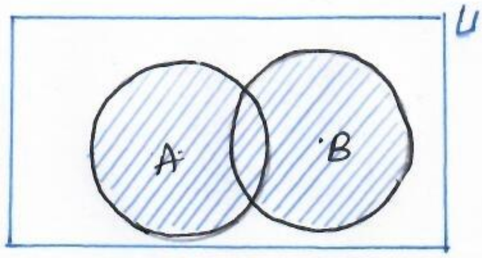
a) Complement laws: (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$

b) De Morgan's law: (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Operations on Sets

1) Union of Sets :

The union of two sets A and B, denoted by $A \cup B$, is defined as the set of those elements which either belong to A or to B or to both.



$A \cup B$

Symbolically,

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Properties : (i) $A \cup B = B \cup A$ (Commutative law)

(ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)

(iii) $A \cup \phi = A$ (Law of identity) ϕ is called identity of $\cup$)

(iv) $A \cup A = A$ (Idempotent law)

(v) $U \cup A = U$ (law of U) or $A \cup U = U$

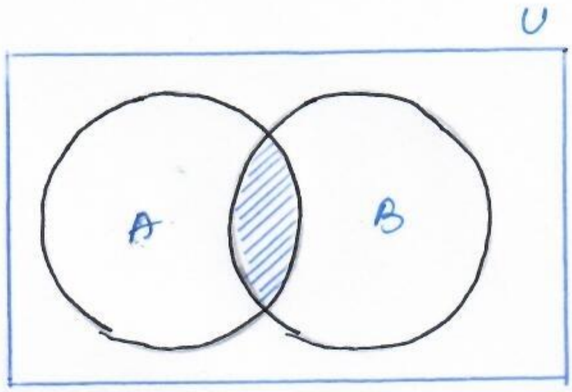
If $A \subset B$, then $A \cup B = B$

For any set A, $A \cup A' = U$

If $A = B$, then $A \cup B = A \cap B$.

2) Intersection of Sets

A and B are any two sets, then the set of common elements of A and B is called the intersection of the sets A and B and is denoted by $A \cap B$.



$A \cap B$

Symbolically,

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

Properties: (i) $A \cap B = B \cap A$ (Commutative law)

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)

(iii) $\phi \cap A = \phi$, $U \cap A = A$ (Law of ϕ and U)
(or) $A \cap U = A$

(iv) $A \cap A = A$ (Idempotent law)

(v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law).

If $A = B$, then $A \cap B = A = B$

If $A \subset B$, then $A \cap B = A$

For any set A, we have $A \cap A' = \phi$.

Disjoint Sets: If A and B are two sets such that $A \cap B = \phi$, their intersection is the null set, then A and B are called disjoint sets.

If $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ $A \cap B = \phi$
A and B are disjoint sets.

Differences of Sets

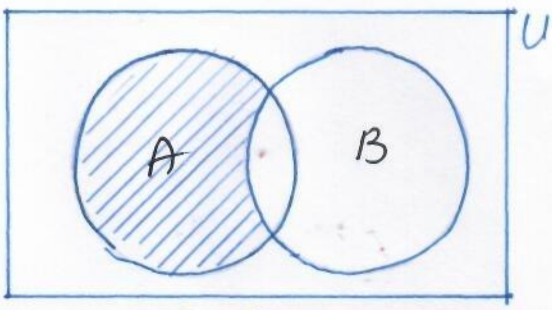
The difference of a set A w.r.t set B is a set which contains only those elements of A which do not belong to B and is denoted by the symbol A−B.

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

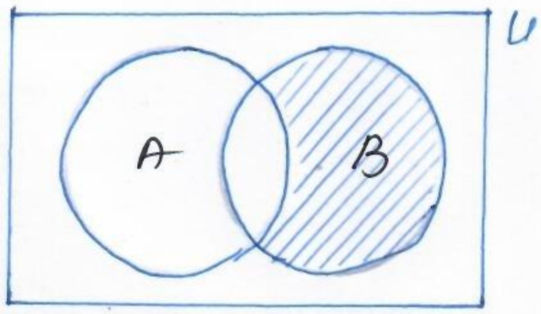
$$B - A = \{x : x \in B \text{ and } x \notin A\}$$

(i) $(A-B) \subset A$ (ii) $(B-A) \subset B$

(iii) $A-B$, $B-A$ and $A \cap B$ are disjoint sets.



$A-B$



$B-A$

Symmetric Difference of Two Sets

The symmetric differences of sets A and B is defined as the union (U) of (A−B) and (B−A) and is denoted by $A \Delta B$

$$A \Delta B = (A-B) \cup (B-A)$$

Also, $A \Delta B = (A \cup B) - (A \cap B)$

(i) $A \Delta B = B \Delta A$ (ii) $A \Delta A = \phi$

(iii) $(A \Delta B) \Delta C = A \Delta (B \Delta C)$

(iv) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Remember

1) Idempotent law : (i) $A \cup A = A$ (ii) $A \cap A = A$

2) Identity law : (i) $A \cup \phi = A$ (ii) $A \cap U = A$

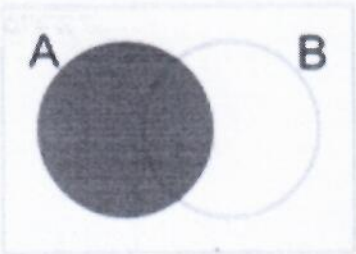
3) Commutative law : (i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

4) Associative law :
(i) $(A \cup B) \cup C = A \cup (B \cup C)$ (ii) $(A \cap B) \cap C = A \cap (B \cap C)$

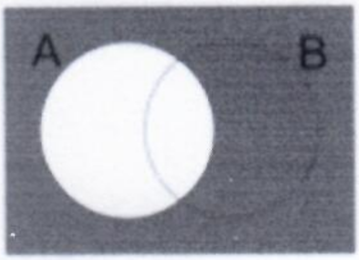
5) Distributive law :
(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6) De-morgan's law : (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$.

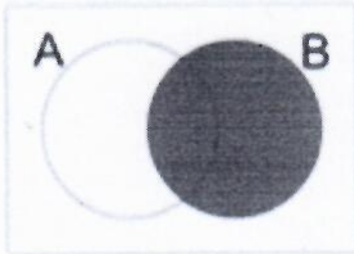
Venn Diagrams



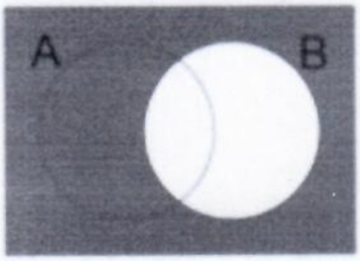
A



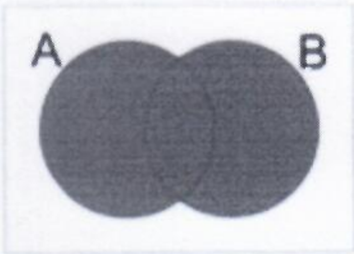
A'



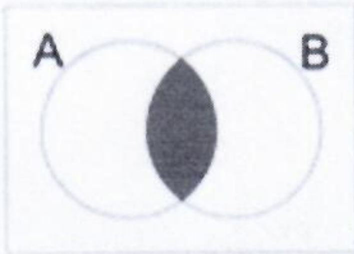
B



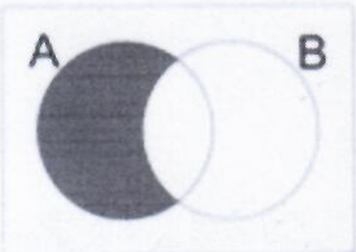
B'



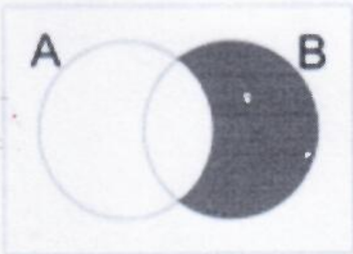
$A \cup B$



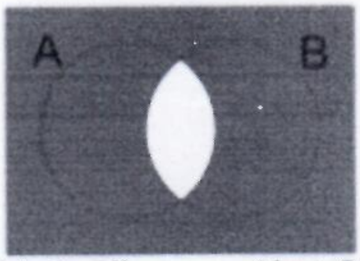
$A \cap B$



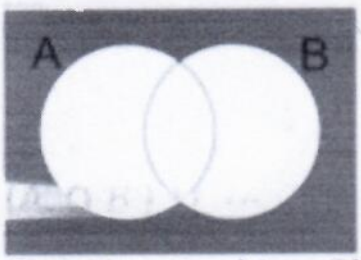
$A \cap B'$



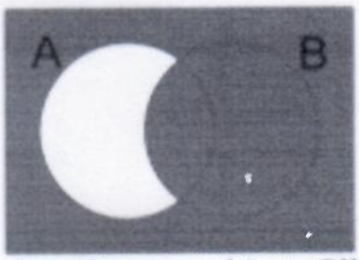
$A' \cap B$



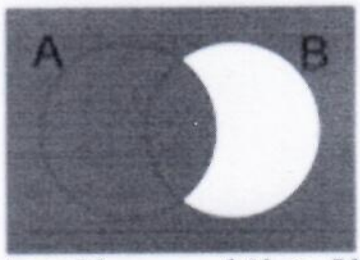
$(A \cap B)'$ or $A' \cup B'$



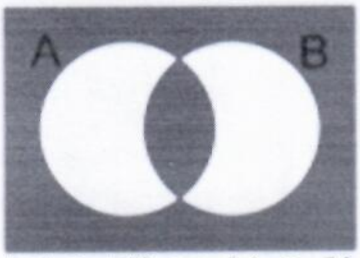
$A' \cap B'$ or $(A \cup B)'$



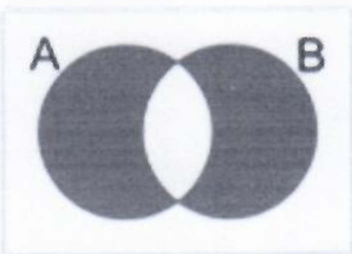
$A' \cup B$ or $(A \cap B')'$



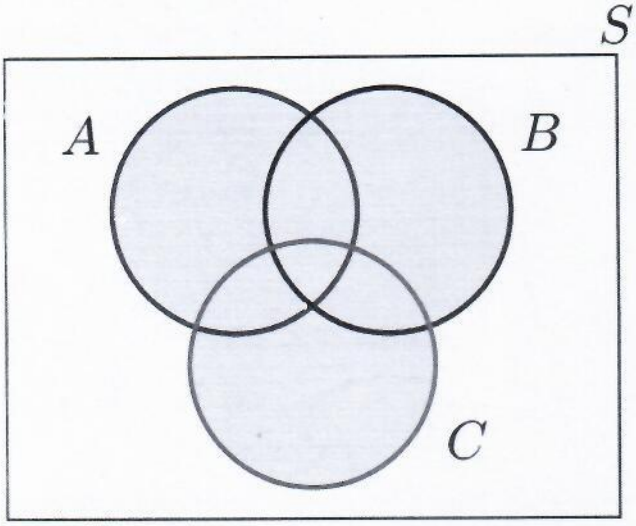
$A \cup B'$ or $(A' \cap B)'$



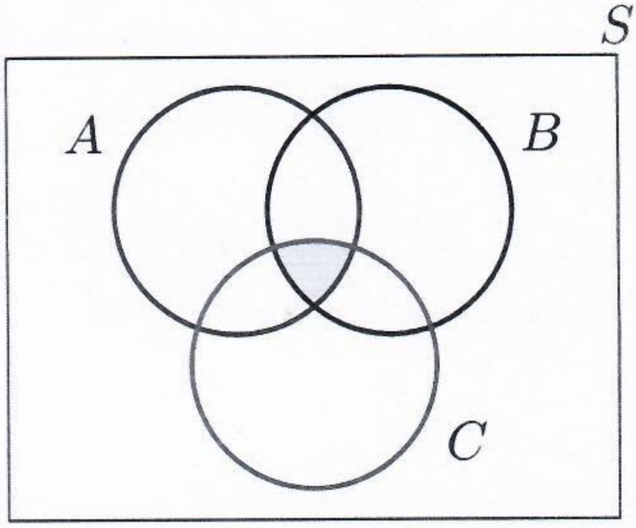
$(A' \cap B') \cup (A \cap B)$



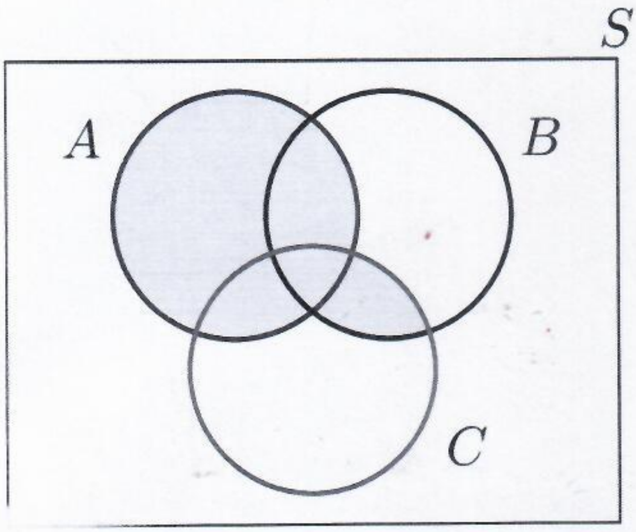
$(A \cap B') \cup (A' \cap B)$



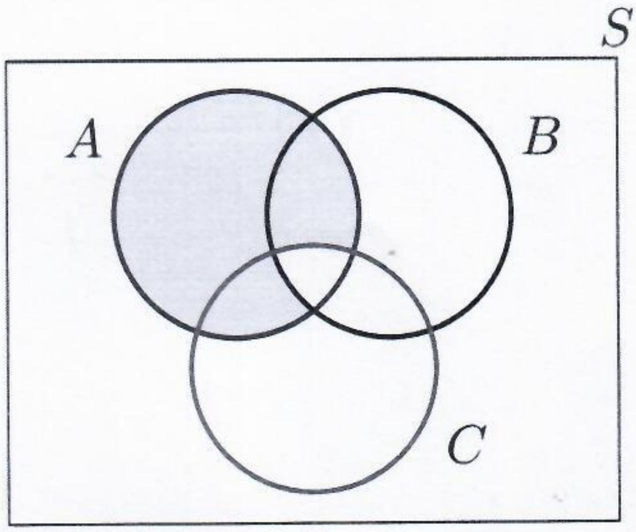
$A \cup B \cup C$



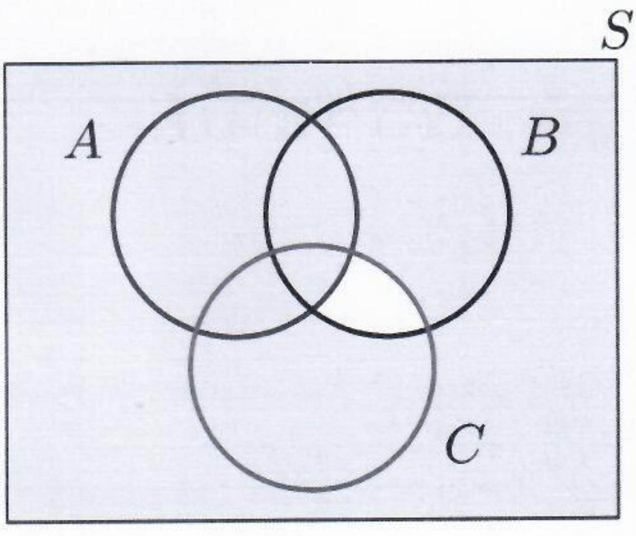
$A \cap B \cap C$



$A \cup (B \cap C)$



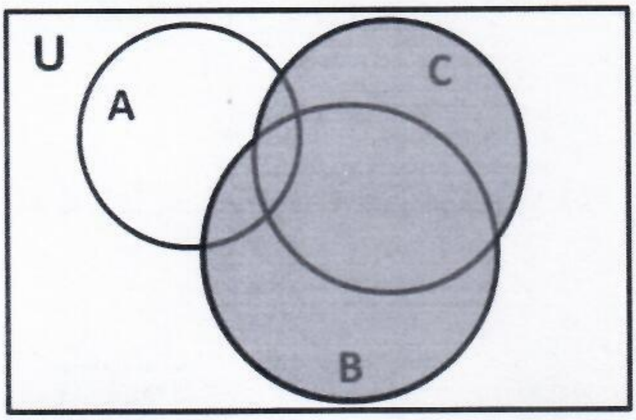
$A - (B \cap C)$



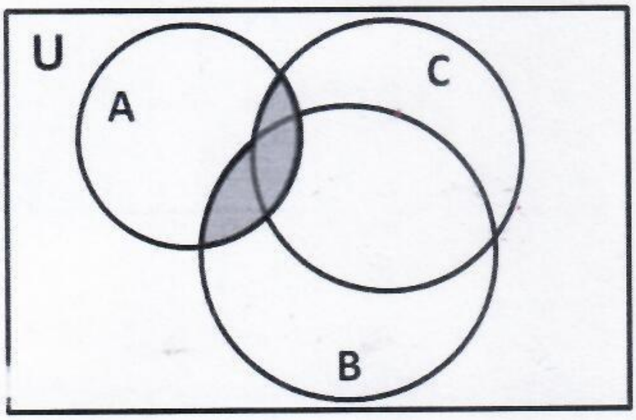
$A \cup (B \cap C)^C$

Distributive law of sets

$A \cap (B \cup C)$

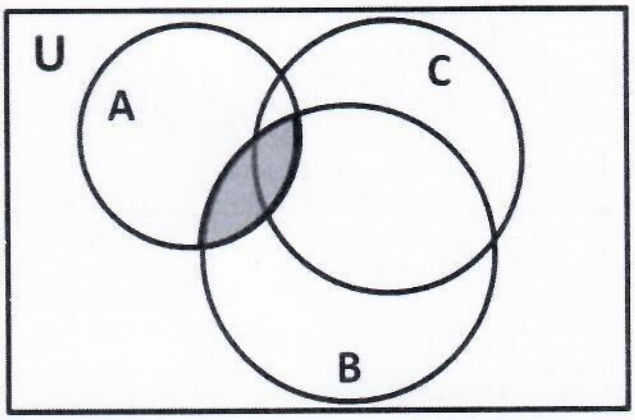


$(B \cup C)$

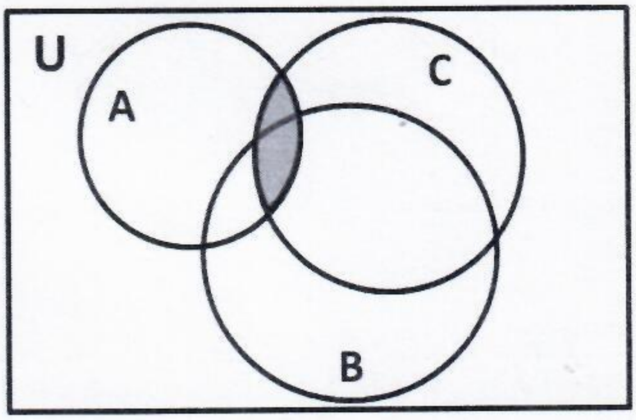


$A \cap (B \cup C)$

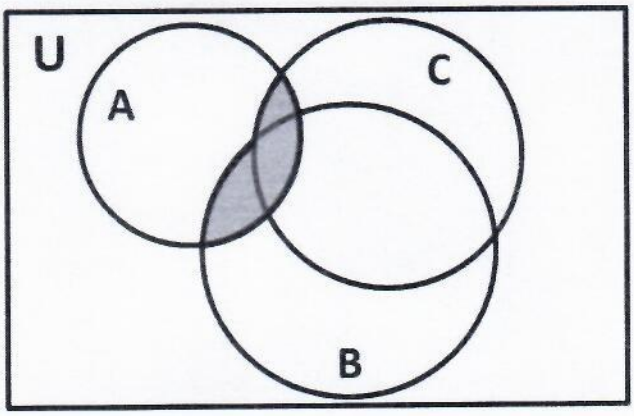
$(A \cap B) \cup (A \cap C)$



$(A \cap B)$



$(A \cap B)$



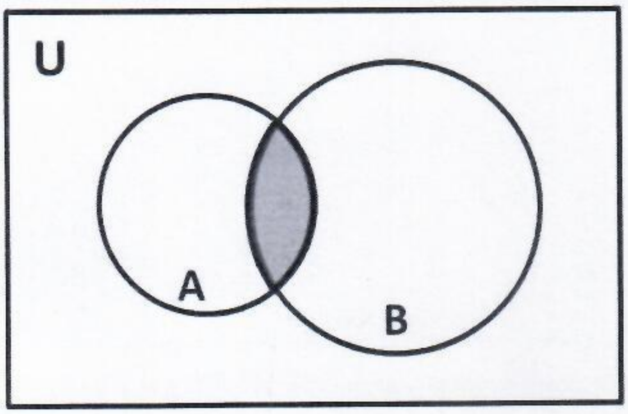
$(A \cap B) \cup (A \cap C)$

$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

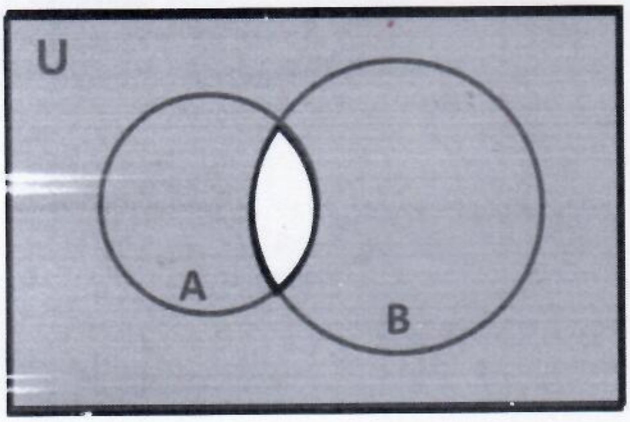
De Morgan's Law

Proving $(A \cap B)' = A' \cup B'$

$(A \cap B)'$

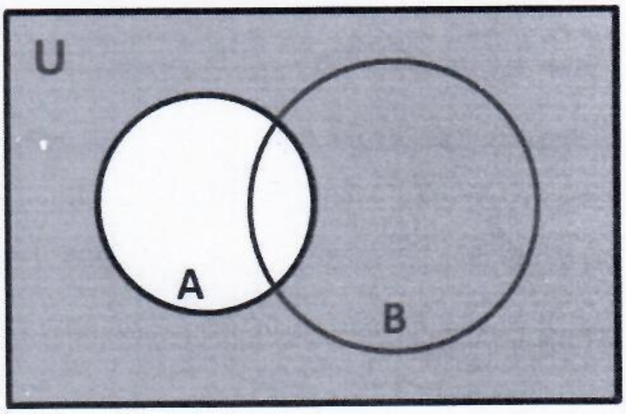


$A \cap B$

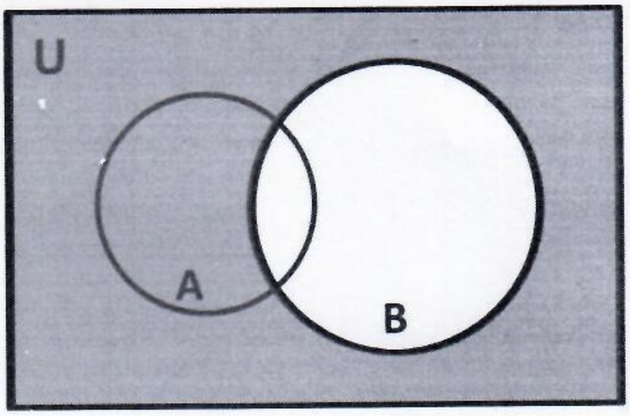


$(A \cap B)'$

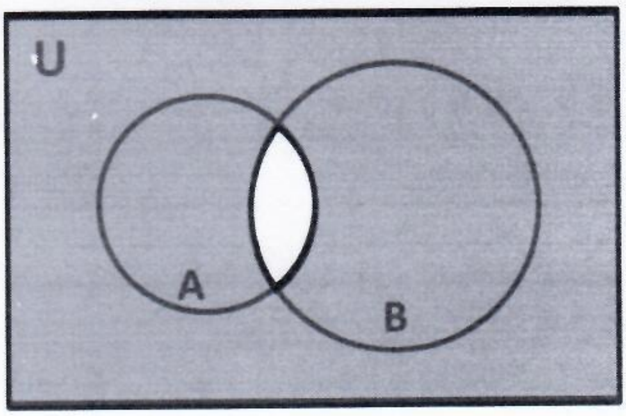
$A' \cup B'$



A'



B'



$A' \cup B'$

$\therefore (A \cap B)' = A' \cup B'$

Solved Examples

(1) State which of the following sets are finite or infinite :

(i) $\{x : x \in N \text{ and } (x-1)(x-2)=0\}$ Finite

(ii) $\{x : x \in N \text{ and } x^2=4\}$ Finite

(iii) $\{x : x \in N \text{ and } 2x-1=0\}$ Finite

(iv) $\{x : x \in N \text{ and } x \text{ is prime}\}$ Infinite

(v) $\{x : x \in N \text{ and } x \text{ is odd}\}$ Infinite.

(2) Find the pairs of equal sets, if any, give reasons :

$A = \{0\}$ $B = \{x : x > 15 \text{ and } x < 5\}$

$C = \{x : x-5=0\}$ $D = \{x : x^2=25\}$

$E = \{x : x$ is an integral positive root of the equation $x^2-2x-15=0\}$

Soln : Only pair of equal sets is C and E

(3) Find if following sets are equal.

$A = \{2,1\}$ $B = \{2,1,1,2,1,2\}$ $C = \{x : x^2-3x+2=0\}$

Soln : $A = \{2,1\}$ $B = \{2,1,1,2,1,2\} = \{1,2\}$

$C = \{x : x^2-3x+2=0\} = \{x : (x-1)(x-2)=0\} = \{1,2\}$

Hence $A = B = C$.

(4) Which of the following pairs of sets are equal? Justify.

(i) X, the set of letters in "ALLOY" and B, the set of letters in "LOYAL".

(ii) $A = \{n : n \in Z \text{ and } n^2 \leq 4\}$ and $B = \{x : x \in R \text{ and } x^2 - 3x + 2 = 0\}$.

Soln: (i) $X = \{A, L, O, Y\} = B$

(ii) $A = \{-2, -1, 0, 1, 2\}$

$x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2) = 0$

$(x-1)(x-2) = 0$

$x = 1, 2$

$\therefore B = \{1, 2\}$

Hence, $A \neq B$.

(5) If $A = \{a, b, c, d, e\}$, find the total number of subsets of A.

Soln: Total number of subsets of $A = 2^n$

$\therefore$ no. of subset $= 2^5 = 32$.

(6) List all the subsets of $\{a, b, c\}$

Soln: The subsets of $\{a, b, c\}$ are:

$\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}$

(7) List the proper subsets of $A = \{1, 2, 3\}$

Soln: $\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}$

(8) If $A = \{2, 4, 6, 8\}$ $B = \{6, 8, 10, 12\}$. find $A \cup B$ & $A \cap B$.

Soln $A \cup B = \{2, 4, 6, 8, 10, 12\}$

$A \cap B = \{6, 8\}$

(9) If $A = \{1, 2, 3, 4, 5, 6\}$ $B = \{2, 4, 6, 8\}$ find $A-B$ and $B-A$.

Soln : $A - B = \{x : x \in A \text{ and } x \notin B\}$

$A - B = \{1, 3, 5\}$ $B - A = \{8\}$

(10) If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{1, 3, 5, 7, 9\}$ find A'.

Soln : $A' = \{x : x \in U \text{ and } x \notin A\} = U - A$

$A' = \{2, 4, 6, 8, 10\}$

(11) If $U = \{1, 2, 3, 4, 5, 6\}$ $A = \{2, 3\}$ $B = \{3, 4, 5\}$
show that $(A \cup B)' = A' \cap B'$.

Soln : $A \cup B = \{2, 4, 3, 5\}$.

$(A \cup B)' = \{1, 6\}$ $A' = \{1, 4, 5, 6\}$ $B' = \{1, 2, 6\}$

$A' \cap B' = \{1, 6\}$

$\therefore (A \cup B)' = A' \cap B'$

Practice Problems

(1) Find the union and intersection of following sets:
$A = \{1, 2, 3, 4\}$ and $B = \{4, 5, 6, 7\}$

(2) Given $A = \{2, 4, 6, 8, 9\}$ $B = \{7, 8, 9, 12\}$ find (i) $A \cup B$ (ii) $A \cap B$

(3) If $A = \{x : x \in N\}$, $B = \{x : x$ is an even number$\}$
$C = \{x : x$ is an odd number$\}$ $D = \{x : x$ is a prime number$\}$

Find (i) $A \cap B$ (ii) $A \cap C$ (iii) $A \cap D$ (iv) $B \cap C$ (v) $B \cap D$ (vi) $C \cap D$.
[$A \cap B = B$, $A \cap C = C$, $A \cap D = D$, $B \cap C = \phi$, $B \cap D = 2$
$C \cap D = \{x : x$ is a prime number greater than $2\}$

(4) If $A = \{x : x \in N$, where x is a multiple of $2\}$
$B = \{x : x \in N$, where x is a multiple of $5\}$
$C = \{x : x \in N$, where x is a multiple of $10\}$

Find $A \cap (B \cup C)$. [$\{10, 20, 30, 40, \ldots\}$

(5) Under what conditions would each of the following be true?
(i) $A \cup B = \phi$ (ii) $A \cup \phi = \phi$ (iii) $A \cap U = U$ (iv) $A \cup \phi = U$
(v) $A \cup B = A \cap B$ (vi) $A' \cap U = U$.

(6) Fill in the blanks:
(i) $A \cap A =$ ___ (ii) $A \cup A' =$ ___ (iii) $\phi' \cap A =$ ___
(iv) $\phi \cup A =$ ___ (v) $A \cap A' =$ ___

(7) Which of the following pairs of sets are disjoint?
(i) $\{1, 2, 3, 4\}$ and $\{x : x$ is a natural number and $4 \leq x \leq 6\}$
(Not disjoint)

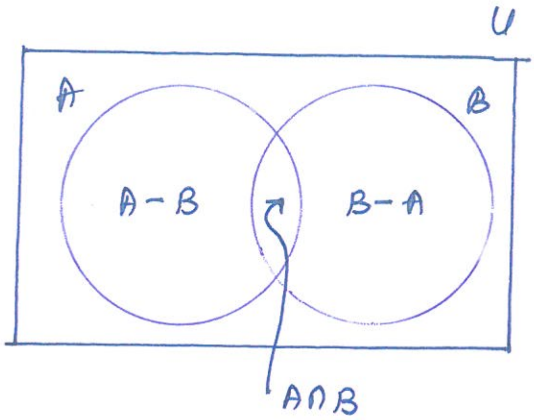
(ii) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$ (Not disjoint)

(iii) $\{x : x$ is an even integers$\}$ and $\{x : x$ is an odd integer$\}$
(Disjoint sets)

Application of Sets

For any two non-empty finite sets A and B, we have

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



We see that $A \cup B$ is divided to three mutually disjoints sets $A - B$, $A \cap B$ and $B - A$.

From above venn diagrame we have

$$n(A) = n(A-B) + n(A \cap B)$$
$$n(B) = n(B-A) + n(A \cap B)$$

Now, $n(A \cup B) = n(A-B) + n(A \cap B) + n(B-A)$

$$n(A \cup B) = [n(A-B) + n(A \cap B)] + [n(B-A) + n(A \cap B)] - n(A \cap B)$$
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

If A and B are disjoint sets, then $A \cap B = \phi$ and $n(A \cap B) = 0$

$$n(A \cup B) = n(A) + n(B).$$

Note:
If A, B and C are any three sets then,

$$n(A \cup B \cup C) = [n(A) + n(B) + n(C)] - [n(A \cap B) + n(B \cap C) + n(A \cap C) + n(A \cap B \cap C)].$$

Remember:

$$n(A') = n(U) - n(A)$$
$$n(A \cap B') = n(A) - n(A \cap B)$$

$$n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$$
$$n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B).$$

Application of Sets

Solved Examples

1) If S and T are two sets such that S has 21 elements, T has 32 elements, and $S \cap T$ has 11 elements, how many elements does $S \cup T$ have?

Sol^n: $n(S) = 21$ $\quad n(T) = 32$ $\quad n(S \cap T) = 11$ $\quad n(S \cup T) = ?$

Using $n(S \cup T) = n(S) + n(T) - n(S \cap T)$

$$= 21 + 32 - 11$$
$$= 42$$

Hence, $S \cup T$ has 42 elements.

2) X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have? [30].

3) In a committe, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak atleast one of these two languages?

Sol^n: People speaking French $n(F) = 50$
People speaking Spanish $n(S) = 10$
People speaking both French and Spanish $n(S \cap F) = 10$

using $n(S \cup F) = n(S) + n(F) - n(S \cap F)$

$$= 20 + 50 - 10$$
$$= 60.$$

(4) In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?

Soln: People like coffee $n(A) = 37$
People like tea $n(B) = 52$

$A \cup B$ = Set of people who like at least one of two drinks.
$A \cap B$ = Set of people who like both the drinks.

$n(A \cup B) = 70$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$70 = 37 + 52 - n(A \cap B)$$
$$n(A \cap B) = 89 - 70 = 19$$

19 people like both coffee and tea

(5) In a school there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics and 4 teach both physics and mathematics. How many teach physics? [12]

(6) In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student like to play at least one of two games. How many students like to play both cricket and football? [5]

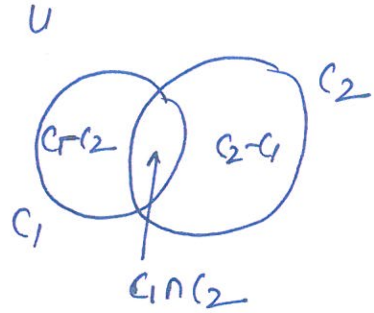
7) In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Find how many students were taking neither apple juice nor orange juice. [225]

Hint. Find $n(A \cup B)'$

8) There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C_1, 50 to chemical C_2 and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to

(i) Chemical C_1 but not chemical C_2
(ii) Chemical C_2 but not chemical C_1
(iii) Chemical C_1 or chemical C_2.

Soln:



(i) $n(C_1 - C_2) = n(C_1) - n(C_1 \cap C_2)$

$n(C_1) = 120 \quad n(C_2) = 50$

$n(C_1 \cap C_2) = 30$

$\therefore n(C_1 - C_2) = 120 - 30$
$= 90$

(ii) $n(C_2 - C_1) = n(C_2) - n(C_1 \cap C_2)$
$= 50 - 30$
$= 20$

(iii) $n(C_1 \cup C_2) = n(C_1) + n(C_2) - n(C_1 \cap C_2)$
$= 120 + 50 - 30$
$= 140.$