CHAPTER 2 : Polynomials

Fundamentals :

- An algebraic equation of the form $p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n$ is called a polynomial, provided it has no negative exponent for any variable, where $a_0, a_1, a_2, ... a_{n-1}, a_n$ are constants (real numbers);
  $a_0 \neq 0$, n is called the degree of the polynomial. (highest power of variable x)
- **Monomial :** Polynomial with one term (ax, by etc.)
- **Binomial :** Polynomial with two terms ($ax + by$, $ax + bx^2$ etc.)
- **Trinomial :** Polynomial with three terms ($ax + by + cz$, $ax + bx^2 + cx^3$ etc.) where $a, b, c \neq 0$.

Degree of a Polynomial :

Degree of a Polynomial : If 1 then polynomial is called **linear polynomial.**
General form : $ax + b$ $(a \neq 0)$
If 2 then polynomial is called **quadratic polynomial.**
General form : $ax^2 + bx + c$ $(a \neq 0)$
If 3 then polynomial is called **cubic polynomial.**
General form : $ax^3 + bx^2 + cx + d$ $(a \neq 0)$

Zeroes of a Polynomial :

Zeroes of a Polynomial : For polynomial $p(x)$, the value of x for which $p(x) = 0$, is called zero(es) of the polynomial, also known as root of the polynomial.

Linear Equations have 1 root.

Quadratic Equations have 2 roots.

Cubic Equations have 3 roots.

Remainder Theorem :

Division Algorithm For Polynomials : If $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then we can find $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$, where, $r(x) = 0$ (or) deg $r(x) <$ deg $g(x)$

Dividend = Divisor × Quotient + Remainder

Factor Theorem : If $p(x)$ is a polynomial of degree ≥ 1 and a is any real number , then

(i) $(x - a)$ is a factor of $p(x)$, if $p(a) = 0$

(ii) $p(a) = 0$, if $(x - a)$ is a factor of $p(x)$.

Algebraic Identities :

(i) $(x + y)^2 = x^2 + 2xy + y^2$

(ii) $(x - y)^2 = x^2 - 2xy + y^2$

(iii) $x^2 - y^2 = (x + y)(x - y)$

(iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(v) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(vi) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

(vii) $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

(viii) $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

(ix) If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

(x) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(xi) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.

Tips :

1. Graph of linear equation is a straight line, while graph of quadratic equation is a parabola.
2. Degree of polynomial = Number of zeroes of polynomial.
3. If remainder $r(x) = 0$, then $g(x)$ is a factor of $p(x)$.
4. $(x + a)$ is a factor of polynomial $p(x)$, if $p(-a) = 0$.
5. $(x - a)$ is a factor of polynomial $p(x)$, if $p(a) = 0$.
6. $(x - a)(x - b)$ is a factor of polynomial $p(x)$, if $p(a) = 0$ and $p(b) = 0$
7. $(ax + b)$ is a factor of polynomial $p(x)$, if $p\left(\frac{-b}{a}\right) = 0$
8. $(ax - b)$ is a factor of polynomial $p(xz)$, if $p\left(\frac{b}{a}\right) = 0$ ■▮