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Chapter

Polynomials

Key Points

1. **Polynomial :** If x is a variable, n is a natural number and $a_0, a_1, a_2, a_3, \ldots\ldots\ldots a_n$ are real numbers, then $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots\ldots + a_1 x + a_0$, $(a_n \neq 0)$ is called a polynomial in x.
2. Polynomials of degree 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.
3. A quadratic polynomial is an algebraic expression of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.
4. Zeroes of a polynomial $p(x)$ are precisely the x – coordinates of the points where the graph of $y = p(x)$ intersects the x–axis, *i.e.,* $x = a$ is a zero of polynomial $p(x)$ if $p(a) = 0$
5. A polynomial can have at most the same number of zeroes as the degree of the polynomial.
6. (*i*) If one zero of a quadratic polynomial $p(x)$ is negative of the other, then co-efficient of $x = 0$

   (*ii*) If zeroes of a quadratic polynomial $p(x)$ are reciprocal of each other, then co-efficient of x^2 = constant term.
7. Relationship between zeroes and coefficients of a polynomial

   If α And β Are zeroes of $p(x)$ $ax^2 + bx + c$ $(a \neq 0)$, them

   Sum of zeroes = $\alpha + \beta = -\frac{b}{a}$

   Product of zeroes = $\alpha\beta = \frac{c}{a}$
8. If α, β are zeroes of a quadratic polynomial $p(x)$, then

   $p(x) = k\,[x^2 - (\text{sum of zeroes})\,x + \text{product of zeroes}]$

   $\Rightarrow\ p(x) = k\,[x^2 - (\alpha + \beta)x + \alpha\beta\ ]$; where k is any non-zero real number.
9. Graph of linear polynomial $p(x) = ax + b$ is a straight line.
10. Division Algorithm states that given any polynomials $p(x)$ and $g(x)$, there exist polynomial $q(x)$ and $r(x)$ such that:

$p(x) = g(x).\ q(x) + r(x)\ ;\ g(x) \neq 0,$

[where either $r(x) = 0$ or degree $r(x)$ < degree $g(x)$]

VERY SHORT ANSWER TYPE QUESTIONS

1. What will be the number of zeroes of a linear polynomial $p(x)$ if its graph (*i*) passes through the origin. (*ii*) doesn't intersect or touch x-axis at any point?

2. Find the quadratic polynomial whose zeroes are

$\left(5+2\sqrt{3}\right)$ and $\left(5-2\sqrt{3}\right)$

3. If one zero of $p(x) = 4x^2 - (8k^2 - 40k)\,x - 9$ is negative of the other, find values of k.

4. What number should be added to the polynomial $x^2 - 5x + 4$, so that 3 is a zero of polynomial so obtained.

5. How many (*i*) maximum (*ii*) minimum number of zeroes can a quadratic polynomial have?

6. What will be the number of real zeroes of the polynomial $x^2 + 1$?

7. If α and β are zeroes of polynomial $6x^2 - 7x - 3$, then form a quadratic polynomial where zeroes are 2α and 2β

8. If α and $\frac{1}{\alpha}$ are zeroes of $4x^2 - 17x + k - 4$, find value of k.

9. What will be the number of zeroes of the polynomials whose graphs are parallel to (*i*) y-axis (ii) x-axis

10. What will be number of zeroes of the polynomials whose graphs are either touching or intersecting the axis only at the points:

(*i*) (–3, 0), (0, 2) & (3, 0) (*ii*) (0, 4), (0, 0) and (0, –4)

SHORT ANSWER TYPE (I) QUESTIONS

11. If –3 is one of the zeroes of the polynomial $(k - 1)x^2 + k\,x + 1$, find the value of k.

12. If the product of zeroes of $ax^2 - 6x - 6$ is 4, find the value of a. Hence find the sum of its zeroes.

13. If α and β are zeroes of the polynomial $x^2 - a(x + 1) - b$ such that $(\alpha + 1)(\beta + 1) = 0$, find the value of b.

14. If zeroes of $x^2 - kx + 6$ are in the ratio 3 : 2, find k.

15. If one zero of the quadratic polynomial $(k^2 + k)x^2 + 68x + 6k$ is reciprocal of the other, find k.

16. If α and β are the zeroes of the polynomial $x^2 - 5x + m$ such that $\alpha - \beta = 1$, find m.

17. If the sum of squares of zeroes of the polynomial $x^2 - 8x + k$ is 40, find the value of k.

18. If α and β are zeroes of the polynomial $t^2 - t - 4$, form a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

SHORT ANSWER TYPE (II) QUESTIONS

19. If $(k + y)$ is a factor of each of the polynomials $y^2 + 2y - 15$ and $y^3 + a$, find values of k and a.

20. Obtain zeroes of $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3}$ and verify relation between its zeroes and coefficients.

21. If $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by $(x^2 + 5)$, remainder comes out to be $(px + q)$, find values of q and q.

22. -5 is one of the zeroes of $2x^2 + px - 15$, zeroes of $p(x^2 + x) + k$ are equal to each other. Find the value of k.

23. Find the value of k such that $3x^2 + 2kx + x - k - 5$ has the sum of zeroes as half of their product.

24. If α and β are zeroes of $y^2 + 5y + m$, find the value of m such that $(\alpha + \beta)^2 - \alpha\beta = 24$

25. If α and β are zeroes of $x^2 - x - 2$, find a polynomial whose zeroes are $(2\alpha + 1)$ and $(2\beta + 1)$

26. Find values of a and b so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$.

27. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$?

28. What must be added to $4x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is divisible by $x^2 - 2x - 3$?

LONG ANSWER TYPE QUESTIONS

29. Find all zeroes of the polynomial $2x^3 + x^2 - 6x - 3$ if two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

30. If $\sqrt{2}$ is a zero of $(6x^3 + \sqrt{2}\,x^2 - 10x - 4\sqrt{2})$, find its other zeroes.

31. If two zeroes of $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $(2 \pm \sqrt{3})$, find other zeroes.

32. On dividing the polynomial $x^3 - 5x^2 + 6x - 4$ by a polynomial $g(x)$, quotient and remainder are $(x - 3)$ and $(-3x + 5)$ respectively. Find $g(x)$

33. If sum and product of two zeroes of the polynomial $x^3 + x^2 - 3x - 3$ are 0 and 3 respectively, find all zeroes of the polynomial.

34. If $-\frac{1}{2}$ is a zero of the polynomial $2x^3 + x^2 - 6x - 3$, find the sum and product of its other two zeroes.

35. Obtain all zeroes of the polynomial $2x^4 - 2x^3 - 7x^2 + 3x + 6$ if two factors of this polynomial are $\left(x \pm \sqrt{\frac{3}{2}}\right)$.

36. Sum and product of two zeroes of $x^4 - 4x^3 - 8x^2 + 36x - 9$ are 0 and -9 respectively. Find the sum and product of its other two zeroes.

37. A person distributes k books to some needy students. If k is a zero of the polynomial $x^2 - 100x - 20000$, then

(*i*) Find the number of books distributed

(*ii*) Which moral values depicted by the person impressed you?

38. One zero of $x^3 - 12x^2 + 47x - 60$ is 3 and the remaining two zeroes are the number of trees planted by two students.

(*i*) Find the total number of trees planted by both students.

(*ii*) Which moral value of the students is depicted here?

ANSWERS

1. (i) 1 (ii) 0

2. $x^2 - 10x + 13$

3. $k = 0, 5$

4. 2

5. (*i*) 2 (*ii*) 0

6. 0

7. $3x^2 - 7x - 6$

8. $k = 8$

9. (*i*) 1 (*ii*) 0

10. (*i*) 2 (*ii*) 1

11. $\frac{4}{3}$

12. $a = -\frac{3}{2}$, sum of zeroes = – 4

13. 1

14. – 5, 5

15. 5

16. 6

17. 12

18. $4t^2 + t - 1$

19. $k = 3, -5$ and $a = 27, -125$

20. $-\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

21. $p = 2, q = 3$

22. $\frac{7}{4}$

23. 1

24. 1

25. $x^2 - 4x - 5$

26. $a = 1$, $b = 7$

27. $14x - 10$

28. $61x - 65$

29. $\sqrt{3}, -\sqrt{3}, -\frac{1}{2}$

30. $-\frac{\sqrt{2}}{2}, -\frac{2\sqrt{2}}{3}$

31. –5, 7

32. $x^2 - 2x + 3$

33. $\sqrt{3}, -\sqrt{3}, -1$

34. 0, 3

35. $2, -1, \pm\sqrt{\frac{3}{2}}$

36. 4, 1

37. (*i*) 200 (*ii*) Love & care, humanity, …..... kindness, etc.

38. (*i*) 9 (*ii*) Love for environment, ..., eco-friendly, etc.