# 5

*Chapter*

# Arithmetic Progression

## Key Points

1. **Sequence**: A set of numbers arranged in some definite order and formed according to some rules is called a sequence.

2. **Arithmetic Progression**: A sequence in which the difference of each term from its succeeding term is constant throughout, is called an arithmetic sequence or arithmetic progression (A.P.).

   In other words A.P. is squence $a_1, a_2, a_3, ........., a_n$ such that $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = .......... a_n - a_{n-1} = d$ and so on.

3. **General Term**: If 'a' is the first term and 'd' is common difference in an A.P., then nth term (general term) is given by $a_n = a + (n - 1)d$.

4. **Sum of n Terms of an A.P.** : If 'a' is the first term and 'd' is the common difference of an A.P., then sum of first n terms is given by

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

   If 'a' is the first term & 'l' is the last/nth term of a finite A.P., then the sum is given by

$$S_n = \frac{n}{2}\{a + l\}$$

5. (i) If $a_n$ is given, then common difference $d = a_n - a_{n-1}$

   (ii) If $S_n$ is given, then nth term is given by $a_n = S_n - S_{n-1}$

   (iii) If $a, b, c$ are in A.P., then $2b = a + c$

   (iv) If a sequence has n terms, its rth term from the end = $(n-r+1)$th term from the beginning.

   (v) Difference of mth and nth term of an A.P. = $(m - n)d$.

## VERY SHORT ANSWER TYPE QUESTIONS

1. Find $5^{th}$ term of an A.P. whose $n^{th}$ term is $3n - 5$.

2. Find the sum of first 10 even numbers.

3. Write the $n^{th}$ term of odd numbers.

4. Write the sum of first n natural numbers.

5. Write the sum of first n even numbers.

6. Find the $n^{th}$ term of the A.P. $-10, -15, -20, -25, ........$

7. Find the common difference of A.P. $4\dfrac{1}{9}, 4\dfrac{2}{9}, 4\dfrac{1}{3}, .........$

8. Write the common difference of an A.P. whose $n^{th}$ term is $a_n = 3n + 7$

9. What will be the value of $a_8 - a_4$ for the following A.P.

   4, 9, 14,........, 254

10. What is value of for the A.P. $-10, -12, -14, -16, ........$

11. If $\dfrac{1}{x+2}, \dfrac{1}{x+3}$ and $\dfrac{1}{x+5}$ are in A.P. find the value of $x$.

12. For what value of $p$, the following terms are three consecutive terms of an A.P.

    $\dfrac{4}{5}, p, 2.$

## SHORT ANSWER TYPE(I) QUESTIONS

13. Is 144 a term of the A.P. 3, 7, 11,...........? Justify your answer.

14. Find the $20^{th}$ term from the last term of the A.P. 3,8,13,.........., 253

15. Which term of the A.P. 5,15,25,..........will be 130 more than its 31st term?

16. The first term, common difference and last term of an an A.P. are 12, 6 and 252 respectively, Find the sum of all terms of this A.P.

17. Find the sum of first 15 multiples of 8.

18. Is the sequence formed in the following sitations an A.P.

    (i) Number of students left in the school auditorium from the total strength of 1000 students when they leave the auditorium in batches of 25.

(ii) The amount of money in the account every year when Rs. 100 are deposited annually to accumulate at compound interest at 4% per annum.

19. Find the sum of even positive integers between 1 and 200.

20. If $4m + 8$, $2m^2 + 3m + 6$, $3m^2 + 4m + 4$ are three consecutive terms of an A.P. find $m$.

21. How many terms of the A.P. 22,20,18,........ should be taken so that their sum is zero.

22. If 10 times of 10th term is equal to 20 times of 20th term of an A.P. find its $30^{th}$ term.

23. Find the middle term of the A.P. 6, 13 , 20, ..........216

24. Which term of the A.P. $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$ is the first negative term? Find the term also.

## SHORT ANSWER TYPE(II) QUESTIONS

25. Find the middle terms of the A.P. 7,13,19,.......,241

26. Find the sum of integers between 10 and 500 which are divisible by 7.

27. The sum of 5th and 9th terms of an A.P. is 72 and the sum of $7^{th}$ and $12^{th}$ term is 97. Find the A.P.

28. If the $m^{th}$ term of an A.P. be $\frac{1}{n}$ and $n^{th}$ term be $\frac{1}{m}$, show that its $(mn)^{th}$ is 1.

29. If the $p^{th}$ of term A.P. is $q$ and the $q^{th}$ term is p, prove that its $n^{th}$ term is $(p + q - n)$.

30. If $p$ times the $p^{th}$ term of an A.P. is equal to $q$ times its $q^{th}$ term, show that the $(p + q)^{th}$ term of the A.P. is zero.

31. For what value of $m$ are the $m^{th}$ terms of the following two A.P.'s the same?

(i) 1,3,5,7,........

(ii) 4,8,12,16,.......

32. The $24^{th}$ term of an A.P. is twice its $10^{th}$ term. Show that $72^{nd}$ term is 4 times its $15^{th}$ term.

33. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

34. If the seventh term of anA.P. is $\frac{1}{9}$ and ninth term is $\frac{1}{7}$, find its $63^{rd}$ term.

35. The sum of 5th and 9th terms of an A.P. is 30. If its 25th term is three times its 8th term, find the A.P.

36. If $S_n$, the sum of first n terms of an A.P. is given by $Sn = 5n^2 + 3n$, then find its $n^{th}$ term and common difference.

## LONG ANSWER TYPE QUESTIONS

37. The sum of third and seventh terms of an A.P. is 6 and their product is 8. Find the sum of first 16th terms of the A.P.

38. If the $m^{th}$ term of an A.P. is $\dfrac{1}{n}$ and the $n^{th}$ term is $\dfrac{1}{m}$, show the sum of its first $(mn)$ terms is $\dfrac{1}{2}(mn + 1)$.

39. If in an A.P. the sum of first m terms is equal to n and the sum of first n terms is m, prove that the sum of first $(m + n)$ terms is $- (m + n)$.

40. Determine the A.P. whase 4th term is 18 and the differemce of 9th trem from the 15th term is 30.

41. If the sum of first $k$ terms of an A.P. is $\dfrac{1}{2}(3k^2 + 7k)$, write its $k^{th}$ term. Hence find its 20th term.

42. The sum of first 9 terms of an A.P. is 162. The ratio of its 6th term to its 13th term is 1:2. Find the first and fifteenth terms of the A.P.

43. If the 10th term of an A.P. is 21 and the sum of its first 10 terms is 120, find its $n^{th}$ term.

44. The sum of first 7 terms of an A.P. is 63 and the sum of its next 7 term is 161. Find the 28th term of this A.P.

45. The sum of first $q$ terms of an A.P. is $63q - 3q^2$. If $p^{th}$ term is $-60$, find the value of $p$. Also find the 11th term of this A.P.

46. In an A.P. the first term is $-2$, the last term is $-29$ and sum of all terms is $-155$. Find the 11th term of this A.P.

47. The sum of first 20 terms of an A.P. is one third of the sum of next 20 term. If first term is 1, find the sum of first 30 terms of this A.P.

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48. The sum of first 10 terms of an A.P. is one third of the sum of next 10 terms. If first term is –5, find the sum of its first 30 terms.

49. The eighth term of an A.P. is half the second term and the eleventh term exceeds one -third of its fourth term by 1. Find its $15^{th}$ term.

50. The sum of first six terms of an A.P. is 42. The ratio of its $10^{th}$ term to its $30^{th}$ term is 1 : 3 calalate the first and thirteenth term of the A.P.

51. An old lady Krishna Devi deposited Rs. 120000 in a bank at 8% interest p.a. She uses the annual interest to give five scholarships to the students of a school for their overall performances each year. The amount of each

    Scholarship is Rs. 300 less than the preceding scholarship. Find the amount of each scholarship. What values of lady are depicted here?

52. Ram asks the labour to dig a well upto a depth of 10 metre. Labour charges are Rs. 150 for first metre and Rs. 50 for each subsequent metre. As labour was uneducated, he claims Rs. 550 for the whole work. What should be the actual amount to be paid to the labour? What value of Ram is depicted in the question if he pays Rs. 600 to the labourer?

# ANSWERS

1. 10
2. 110
3. $2n - 1$
4. $\dfrac{n(n+1)}{2}$
5. $n(n+1)$
6. $-5(n+1)$
7. $\dfrac{1}{9}$
8. 3
9. 20
10. $-40$
11. $x = 1$
12. $\dfrac{7}{5}$
13. No Because $a = 3$ (odd number), $d = 4$ (even number), so each term of the given A.P. will be an odd number.
14. 158
15. $44^{th}$
16. 5412
17. 540
18. (i) Yes (ii) No
19. 9900
20. $m = 0, 2$
21. 23
22. 0
23. 111
24. $28^{th}, -\dfrac{1}{4}$
25. 121, 127
26. 17885
27. 6, 11, 16, 21, 26,.....
31. No such value m exists
33. 89
34. 1
35. 3, 5, 7, 9, 11,.........
36. $a_n = 10n - 2, d = 10$
37. 76, 20
40. 3, 8, 13,.....
41. $a_{20} = 62, a_k = 3k + 2$
42. 6, 48
43. $2n + 1$
44. 57
45. $p = 21, a_{11} = 0$
46. $-32$
47. 900
48. $-4500$
49. 3
50. First term = 2
    $13^{th}$ term = 26
51. Rs. 2520, Rs. 2220, Rs. 1920, Rs. 1620, Rs. 1320 Love charity etc
52. Rs. 600, Honesty, Sincerity