Chapter 7: Permutations and Combinations (XI)

Introduction

Suppose we have a suitcase with a number lock which has 4 wheels labelled 0 to 9. The lock can be opened if 4 specific digits are arranged in a particular sequence with no repetition. You remember only first digit which is 7. In order to open the lock, how many sequences of 3-digits you may have to check with? We start listing all possible arrangements of 9 remaining digits taken 3 at a time.

Factorial Notation

The continued product of first n natural numbers is called factorial n and is denoted by the symbol $n!$

Thus $n! = 1\cdot 2\cdot 3 \cdots (n-2)\cdot(n-1)\cdot n$

or

$= n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$ (writing in reverse order)

$6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 6\cdot5\cdot4\cdot(3!)$

We define $0! = 1$

Note: We don't define factorial of proper fractions and negative numbers. Rather factorial is defined only for whole numbers.

Groups and Arrangements

Consider three letters a, b, c

The no. of groups taken all at a time = 1 viz abc

The no. of arrangements taken all at a time = 6 viz abc, acb, bca, bac, cab, cba

The no. of groups taken two at a time = 3 viz ab, bc, ca

The no. of arrangements taken two at a time = 6 viz ab, ba, bc, cb, ca, ac.

Fundamental Principle of Counting

Mohan has 3 pants and 2 shirts. How many different pairs of a pant and a shirt, can he dress up with?
There are three ways in which pant can be chosen. Similarly, a shirt can be chosen in 2 ways. For every choice of a pant there are 2 choices of a shirt. Therefore, there are $3 \times 2 = 6$ pairs of a pant and a shirt.

Sabnam has 2 school bags, 3 tiffin boxes and 2 water bottles. A shool bag can be choosen in 2 ways. After a bag is chosen, a tiffin box can be chosen in 3 different ways. Hence, there are $2 \times 3 = 6$ pairs of shool bag and a tiffin box. For each of these pairs a water bottle can be chosen in 2 different ways. Hence, there are $6 \times 2 = 12$ different ways in which Sabnam can carry these items to shool.

Multiplication Principle, which states that:
"If an even can occur in m different ways, following which other even can occur in n different ways, then the total number of occurrence of the events in the given order is $m \times n$."

Addition Principle: If an even can occur in m different ways and a second even in n different ways, then either of the two events can occur in $(m+n)$ ways provided only one event can occur at a time.

Example 1: Find the number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE, where the repetition of the letters is not allowed.

Solution: Consider 4 vacant places __ __ __ __ .

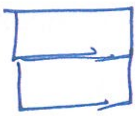
The first place can be filled with 4 different ways by any of 4 letters. Second place can be filled by anyone of the remaining 3 letters in 3 different ways. Similarly third can be filled by remaining 2 letters and fourth place can be filled in 1 way.

Thus, the number of ways in which the 4 places can be filled, by the multiplication principle, is $4 \times 3 \times 2 \times 1 = 24$.

Hence, the required number of words is 24.

Example 2: Given 4 flags of different colours, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?

Sol^n: Consider 2 vacant places



The upper vacant place can be filled in 4 different ways by anyone of the 4 flags. Lower vacant place can be filled in 3 different ways by anyone of remaining 3 different flags.

Hence, by the multiplication principle, the required number of signals $= 4 \times 3 = 12$.

Example 3: How many two digit even numbers can be formed from the digits 1, 2, 3, 4, 5 if the digits can be repeated?

Sol^n: Consider two vacant places $\underline{T}$ $\underline{U}$.

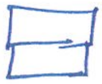
Unit place can be filled in 2 ways (2 & 4)

Tens place can be filled in 5 ways (all digits)

Hence, required two digit numbers will be $2 \times 5 = \underline{10}$

Example 4: Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available.

Solution: A signal consist of of either 2 flags, 3 flags, 4 flags or 5 flags.

2 Flag signal  : There are 5 flags. First place can be filled in 5 ways and second place can be filled in 4 ways.

The total two places can be filled up in $5 \times 4 = 20$ ways.

Thus no. of 2 flag signals = 20.

Similarly, no. of 3 flag signals $= 5 \times 4 \times 3 = 60$

No. of 4 flag signals $= 5 \times 4 \times 3 \times 2 = 120$

No. of 5 flag signals $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Hence, required no. of signals $= 20 + 60 + 120 + 120$

$= 320$

Example 5: How many numbers are there between 100 and 1000 such that every digit is either 2 or 9?

Soln: Each number between 100 and 1000 consists of three digit. — — —.

Each place can be filled with 2 ways (2 or 9).

Hence, total no. of numbers $= 2 \times 2 \times 2 = \underline{8}$

Permutations

The different arrangements, which can be made out of a given number of things by taking some or all at a time, are called permutations.

Permutations when all the objects are distinct

Theorem 1: The number of permutation of n different objects taken r at a time, where $0 < r \leq n$ and the objects do not repeat is $n(n-1)(n-2)\cdots(n-r+1)$, which is denoted by ${}^nP_r$.

$${}^nP_r = n(n-1)(n-2)\ldots\ldots(n-r+1) = \frac{n!}{(n-r)!} \qquad 0 \leq r \leq n$$

In particular, when $r = n$, ${}^nP_n = \frac{n!}{0!} = n!$

Counting permutations is merely counting the number of ways in which some or all objects at a time are rearranged.

When $r = 0$ $\quad {}^nP_0 = 1 = \frac{n!}{n!} = \frac{n!}{(n-0)!}$.

Theorem 2: The number of permutation of n different objects taken r at a time, where repetition is allowed is n^r.

$${}^nP_r = {}^{n-1}P_r + r\,{}^{n-1}P_{r-1}$$

Permutations when all the objects are not distinct objects.

Theorem 3: The number of permutations of n objects, where p objects are of same kind and rest are all different.

$$= \frac{n!}{p!}$$

Theorem 4: The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and rest, if any, are different kind

$$= \frac{n!}{p_1!\, p_2! \ldots p_k!}$$

Example 1: Find the number of permutations of the letters of the word ALLAHABAD.

Soln: There are 9 Letters. A's - 4 Nos L's - 2 Nos.

Required number of arrangements $= \dfrac{9!}{4!\,2!} = \dfrac{9 \cdot 8 \cdot 7 \cdot \overset{3}{\cancel{6}} \cdot 5 \cdot \cancel{4!}}{\cancel{4!}\; \cancel{2} \cdot 1} = 7560$

Example 2: How many 4-digit numbers can be formed by using the digits 1 to 9 if repetition of digits is not allowed?

Soln: Permutations of 9 different digits taken 4 at a time.

Therefore, the required 4 digit numbers $= {}^9P_4 = \dfrac{9!}{(9-4)!} = \dfrac{9!}{5!}$

$= 9 \times 8 \times 7 \times 6$

$= 3024.$

Example 3: How many numbers lying between 100 and 1000 can be formed with the digits 0, 1, 2, 3, 4, 5, if the repetition of digits is not allowed?

Soln: Every number between 100 and 1000 is a 3-digit. We first have to count the permutations of 6-digits taken 3 at a time.
This number would be ${}^6P_3$.

But these permutation will include those also where 0 is at the 100's place, viz 092, 042, ... which are actually 2-digit numbers. Hence such numbers has to be substracted.
To get permutations of such numbers, we fix 0 at the 100's place and rearrange the remaining 5 digits taking 2 at a time.
This number is ${}^5P_2$.

Therefore, required number $= {}^6P_3 - {}^5P_2 = \frac{6!}{3!} - \frac{5!}{3!}$

$= 6\times5\times4 - 5\times4$

$= 120 - 20$

$= 100$

Example 4: Find the value of n such that
(i) ${}^nP_5 = 42\,{}^nP_3 \quad n>4$ (ii) $\frac{{}^nP_4}{{}^{n-1}P_4} = \frac{5}{3} \quad n>4.$

Soln: (i) ${}^nP_5 = 42\,{}^nP_3$

$\cancel{n}\cancel{(n-1)}\cancel{(n-2)}(n-3)(n-4) = 42\,\cancel{n}\cancel{(n-1)}\cancel{(n-2)}$

Since $n>4$, so $n(n-1)(n-2) \neq 0$

$(n-3)(n-4) = 42$

$(n-3)(n-4) - 42 = 0$; $n^2 - 7n + 12 - 42 = 0$

$\Rightarrow n^2 - 7n - 30 = 0$ $\qquad 30 = 10 \times 3$

$n^2 - 10n + 3n - 30 = 0$

$n(n-10) + 3(n-10) = 0$

$(n+3)(n-10) = 0$

$n = -3, +10.$

n cannot be negative, so $n = 10$.

(ii) $\dfrac{{}^{n}P_4}{{}^{n-1}P_4} = \dfrac{5}{3}$ $\quad n > 4 \Rightarrow 3 \cdot n \cancel{(n-1)}\cancel{(n-2)}\cancel{(n-3)}$
$= 5[\cancel{(n-1)}\cancel{(n-2)}\cancel{(n-3)}(n-4)]$

$\Rightarrow 3 \times n = 5(n-4)$

$3n = 5n \cdot 20$; $2n = 20$, $\underline{\underline{n = 10}}$

Example 5: Find r, if $5\,{}^{4}P_r = 6\,{}^{5}P_{r-1}$

Sol^n: $5\,{}^{4}P_r = 6\,{}^{5}P_{r-1}$

$$5 \times \frac{4!}{(4-r)!} = 6\,\frac{5!}{(5-r+1)!}$$

$$\frac{5!}{(4-r)!} = \frac{6 \times 5!}{(5-r+1)(5-r+1-1)(5-r+1-2)!}$$

$$\frac{\cancel{5!}}{\cancel{(4-r)!}} = \frac{6 \times \cancel{5!}}{(6-r)(5-r)\cancel{(4-r)!}}$$

$(6-r)(5-r) = 6$

$r^2 - 11r + 24 = 0$

$r^2 - 8r - 3r + 24 = 0$

$(r-8)(r-3) = 0$

$r = 8$ or $r = 3$.

Here $r = \underline{\underline{8, 3}}$

Example 6 : Find the number different 8-letter arrangements that can be made from the letters of the word DAUGHTER so that (i) all vowels occurs together
(ii) all vowels do not occur together.

Soln : vowels - a, e, i, o, u.
Vowels in daughter word, - a, e, u (3 vowels)

(i) Since vowels occurs together, assume (a, e, u) as one object. This single object with remaining 5 objects will be counted as 6 objects.

Permutation of these 6 objects taken all at a time $= {}^6P_6$
$= 6!$
We shall have have 3! permutations of three vowels a, e, u.

Hence, by multiplication principle the required number of permutation $= 6! \times 3! = 4320$.

(ii) All vowels are never together.

Required number = possible arrangements of 8 letters taken all at a time — permutations in which vowels are always together.
$= 8! - 6! * 3!$
$= 6!(7 \times 8 - 6)$
$= 50 \times 6!$
$= 50 \times 720$
$= 36000$

Example 7: In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if the discs of the same colour are indistinguishable?

Soln: Total number of discs $= 4 + 3 + 2 = 9$

the number of arrangements $= \dfrac{9!}{4!\,3!\,2!} = 1260$.

Example 8: Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,

(i) do the words start with P (ii) do all the vowels always occur together

(iii) do the vowels never occur together

(iv) do the words begin with I and end in P?

Soln: There are 12 letters N - 3, E - 4, D - 2 times.

Therefore, required number of arrangements $= \dfrac{12!}{3!\,4!\,2!} = 1663200$

(i) Let's fix P at extreme left position.

Therefore, the required words starts with P $= \dfrac{11!}{3!\,2!\,4!} = 138600$

(ii) There are 5 vowels - 4 Es, 1 I

We treat vowels as single objects since they occur together.

Total objects - $7+1 = 8$, 3 Ns, 2 Ds

No. of arrangements $= \dfrac{8!}{3!\,2!}$

Five vowels E, E, E, E and I can be arranged in $= \dfrac{5!}{4!}$

By multiplication principle required arrangement

$= \dfrac{8!}{3!\,2!} \times \dfrac{5!}{4!} = 16800$.

(iii) The required number of arrangements
= the total number of arrangements − the number of of arrangements where all the vowels occur together.

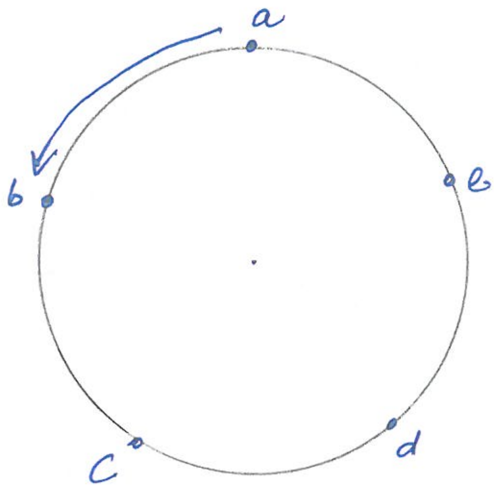
= 1663200 − 16800
= 1646400

(iv) Lets fix J and P at the extreme ends
We are left with 10 letters.

Required number of arrangements $= \dfrac{10!}{3!\,2!\,4!} = 12600$.

CIRCULAR PERMUTATIONS

If we consider arrangements of objects in the form of circle, instead of a line, such permutation are called circular permutation.



Example, Let 5 persons be seated around a round table.

In the case of round table, there is no distinction between the first place and last place. Here only the changes in the relative places are considered.

Let any one of the n objects be fixed at the first place. Now remaining $(n-1)$ objects can be arranged amoung themselves in ${}^{n-1}P_{n-1}$ i.e. $(n-1)!$ ways.
Hence, required number of ways $= (n-1)!$

If $n = 5$, required number of ways $= (5-1)!$

Cor. Clockwise and Anti-clockwise Arrangements $= \frac{1}{2}(n-1)!$

Example 9: In how many ways can 8 student be seated in : (i) circle (ii) a line ?

Soln : (i) In circle $= (8-1)! = 7! = 5040$

(ii) In line $= 8! = 40320$

Example 10: In how many ways can 6 beads of same colour form a necklace ?

Soln : Here the no. of circular permutation 6 bead taken all at a time $= (6-1)!$ i.e. $5!$

But in case of necklace, there is no difference between clockwise and anti-clockwise permutation.

$\therefore$ reqd no. of ways $= \frac{1}{2}(5!) = \frac{1}{2} \times 120 = 60$.

Example 11: In how many ways can 7 person sit at a round table so that all shall not have the same neighbour in any two arrangements.

Soln : 7 person can sit in round table in $6!$ ways

But in clockwise and anti-clockwise arrangement, each person shall have the same neighbour.

Hence required number of ways $= \frac{1}{2}(6!)$

$= \frac{1}{2}(720)$

$= 360$.

Combinations

There is a group of 3 lawn tennis players X, Y, Z.
A team consisting of 2 players is to be formed.
Here the team of X and Y is not different from team of Y and X. Here, order is not important.
There are only 3 possible ways in which the team could be constructed. / XY, YZ and ZX.

Here, each selection is called a combination of 3 different objects taken 2 at a time.
In a combination, the order is not important.

Theorem 5: $^nP_r = {}^nC_r \, r!$ $\quad 0 < r \leq n$

Remarks: (1) $\dfrac{n!}{(n-r)!} = {}^nC_r \cdot r! \Rightarrow {}^nC_r = \dfrac{n!}{r!(n-r)!}$

If $r = n$ $\quad {}^nC_n = \dfrac{n!}{n!\,0!} = 1$

(2) We define ${}^nC_0 = 1$ i.e. the number of combinations of n different things taken nothing at all is considered to be 1.

(3) Since ${}^nC_0 = 1$ $(r=0)$ $\therefore$ ${}^nC_r = \dfrac{n!}{r!(n-r)!}$ $\quad 0 \leq r \leq n$.

(4) ${}^nC_{n-r} = \dfrac{n!}{(n-r)!(n-(n-r))!} = \dfrac{n!}{(n-r)!\,r!} = {}^nC_r$

i.e. Selecting r objects out of n objects is same as rejecting $(n-r)$ objects.

(5) ${}^nC_a = {}^nC_b \Rightarrow a = b$ or $a = n-b$; $n = a+b$

Theorem 6: ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Example 1: $^nC_9 = {}^nC_8$, find n.

Soln : $\frac{n!}{(n-9)!\,9!} = \frac{n!}{(n-8)!\,8!}$; $\frac{\cancel{n!}}{\cancel{(n-9)!}\,9\times\cancel{8!}} = \frac{\cancel{n!}}{(n-8)\,\cancel{(n-9)!}\,\cancel{8!}}$

$\frac{1}{9} = \frac{1}{n-8}$

$n-8 = 9$

$n = 17$

(or) If $^nC_a = {}^nC_b$

$n = a+b$

$\therefore n = 9+8$

$= 17$.

Example 2: A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?

Soln: Here, order does not matter. We need to count combinations.

combinations of 5 different persons taken 3 at a time

$= {}^5C_3$

$= \frac{5!}{(5-3)!\,3!} = \frac{5!}{2!\,3!} = \frac{5\cdot \overset{2}{\cancel{4}}\cdot \cancel{3!}}{\cancel{2}\times \cancel{3!}} = 10$

Now, 1 man can be selected from 2 men in $^2C_1$ ways.
2 women can be selected from 3 women in $^3C_2$ ways.

$\therefore$ the required number of committees $= {}^2C_1 \times {}^3C_2$

$= \frac{2!}{1!\,1!} \times \frac{3!}{2!\,1!}$

$= 6$

Example 3: What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these

(i) four cards are of same suit

(ii) four cards belong to four different suits

(iii) are face cards.

(iv) Two are red cards and two are black cards.

(v) cards are of the same colour?

Soln: Combinations of 52 different things, taken 4 at a time

$$= {}^{52}C_4 = \frac{52!}{(52-4)!\,4!} = \frac{52!}{48!\,4!}$$

$$= \frac{52\cdot 51\cdot 50\cdot 49\cdot \cancel{48!}}{1\cdot 2\cdot 3\cdot 4\times \cancel{48!}}$$

$$= 270725$$

(i) There are four suits : diamond, club, spade and heart and 13 cards of each suit.

There are ${}^{13}C_4$ ways of choosing each suit.

$\therefore$ The required number of ways $= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$

$$= 4\times \frac{13!}{(13-4)!\,4!} = \frac{4\times 13!}{9!\times 4!}$$

$$= \frac{\cancel{4}\times 13\cdot 12\cdot 11\cdot 10\cdot \cancel{9!}}{\cancel{9!}\; 1\cdot 2\cdot 3\cdot \cancel{4}}$$

$$= 2860$$

(ii) There are ${}^{13}C_1$ ways of choosing 1 card from each suit.

Hence, by multiplication principle,

required number of ways $= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

$$= 13^4.$$

(iii) There are 12 face cards and 4 are to be selected out of these 12 cards. This can be done in $^{12}C_4$ ways.

$$^{12}C_4 = \frac{12!}{(12-4)!\,4!} = 495.$$

(iv) There are 26 red cards and 26 black cards.

Required number of ways $= {}^{26}C_2 \times {}^{26}C_2$

$$= \left[\frac{26!}{2!\,24!}\right]^2$$

$$= (325)^2$$

$$= 105625$$

(v) 4 red cards can be selected out of 26 red cards on $^{26}C_4$ ways. 4 black cards can be selected out of 26 cards in $^{26}C_4$ ways.

$\therefore$ required number of ways $= {}^{26}C_4 + {}^{26}C_4$

$$= \frac{2 \times 26!}{(26-4)!\,4!} = \frac{2 \times 26!}{22!\,4!}$$

$$= 29900.$$

Example 4: If $^nC_7 = {}^nC_5$, find $^nC_4$

We know that if $^nC_a = {}^nC_b$ then $n = a+b$

$\therefore \; n = 7+5 = 12$

$$\therefore \; ^nC_4 = {}^{12}C_4 = \frac{12!}{(12-4)!\,4!} = \frac{12!}{8!\,4!} = \frac{\cancel{12}^{\,}\cdot 11 \cdot \cancel{10}^{5} \cdot 9 \cdot \cancel{8!}}{1\cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4} \times \cancel{8!}}$$

$$= 11 \times 5 \times 9$$

$$= 495$$

Miscellaneous Examples.

(1) How many words, with or without meaning, each of 3 vowels and 2 consonants can be formed fom the letters of the word INVOLUTE?

Soln: INVOLUTE ; Vowels - I, O, U, E (4 Nos)
Consonants - N, V, L, T (4 Nos)

The number of ways of selecting 3 vowels out of 4 $= {}^4C_3 = 4$.

The number of ways of selecting 2 constants out of 4 $= {}^4C_2 = 6$.

$\therefore$ the number of combinations of 3 vowels and 2 consonants
$= 4 \times 6 = 24$.

(2) A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members by selecting if the team has (i) no girl? (ii) at least one boy and one girl? (iii) at least 3 girls?

Soln: (i) Since no girls can be selected.
5 boys out of 7 boys can be selected in ${}^7C_5$ ways.

$$= {}^7C_5 = \frac{7!}{5!\,2!} = 21$$

(ii) Since, at least one boy and one girl are to be there in every team. Therefore, the team can consists of

(a) 1 boy and 4 girls can be selected in ${}^7C_1 \times {}^4C_4$ ways
(b) 2 boys and 3 girls can be selected in ${}^7C_2 \times {}^4C_3$ ways
(c) 3 boys and 2 girls can be selected in ${}^7C_3 \times {}^4C_2$ ways
(d) 4 boys and 1 girl can be selected in ${}^7C_4 \times {}^4C_1$ ways

$\therefore$ Required number of ways

$$= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$$
$$= 7 + 84 + 210 + 140 = 441$$

(iii) at least 3 girls, the team can consist of

3 girls and 2 boys can be selected in ${}^4C_3 \times {}^7C_2$ ways
4 girls and 1 boys can be selected in ${}^4C_4 \times {}^7C_1$ ways

$\therefore$ Required number of ways

$$= {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$$
$$= 84 + 7$$
$$= 91$$

(3) Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in a dictionary, what will be the 50^{th} word?

Soln: There are 5 letters in word AGAIN in which A appear 2 times.

Required number of words $= \dfrac{5!}{2!}$
$= 60$

No. of words starting with A (we fix the letter A)
$\therefore$ No. of words starting A $= 4! = 24$.

No. of words starting with G $= \dfrac{4!}{2!} = 12$

No. of words starting with I $= \dfrac{4!}{2!} = 12$

Total number of words obtained so far $= 24 + 12 + 12$
$= 48$

49^{th} word is NAAGI
50^{th} word is NAAIG.

(4) How many numbers greater than 1000000 can be formed by using the digit 1, 2, 0, 2, 4, 2, 4 ?

Soln: 1000000 is a 7 digit number

Since the number have to be greater than 1000000, so they can begin either with 1, 2 or 4.

The number of numbers beginning with 1 = $\frac{6!}{3!\,2!} = \frac{4\times5\times6}{2}$
(as 1 is fixed at extreme left position)

$= 60$

Total numbers begining with 2 = $\frac{6!}{2!\,2!} = \frac{3\times4\times5\times6}{2} = 180$
(fix one 2 at extreme left)

Total numbers begining with 4 = $\frac{6!}{3!} = 4\times5\times6 = 120$

∴ required number of numbers = $60 + 180 + 120$
$= 360$.

Alternative Method

The number of 7 digit arrangements = $\frac{7!}{3!\,2!} = 420$

The number of arrangement starting 0 at extreme left
$= \frac{6!}{3!\,2!} = 60$.

Required numbers = $420 - 60$
$= 360$.

(5) In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?

Soln: Let us first seat 5 girls. This can be done in 5!.

× G × G × G × G × G ×

There are 6 cross marks where boys can seat.
Three boys can be seated in $^6P_3$ ways.
By multiplication principle = $5! \times {}^6P_3 = 5!\,\frac{6!}{3!}$
$= 4\times5\times2\times3\times4\times5\times6$
$= 14400$

(6) In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together.

Soln: ASSASSINATION – 13 letters.

As – 3 times Ss – 4 times Is – 2 times

Ns – 2 times

Treat SSSS as a single objects.

This single object together with remaining 9 object will account for 10 objects.

Required number of ways of arranging the letters of given word (3 As, 2 Ns, 2 Is) $= \dfrac{10!}{3!2!2!} = 151200$

(7) From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can be excursion party be chosen?

Soln: Case 1: All three students join.

The remaining 7 students can be chosen from remaining 22 students in $^{22}C_7$ ways.

Case II: None of the three students join.

10 students can be chosen from remaining 22 students in $^{22}C_{10}$ ways.

Required number of ways $= {}^{22}C_7 + {}^{22}C_{10}$.

(8) It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

Soln: The 5 men can be seated in 5! ways

5 × 5 × 5 × 5 × 5

4 women can be seated as crossed place in 4! ways.

Possible no. of arrangements $= 5! \times 4! = 24 \times 120 = 2880$.