Mathematics (X)

Chapter 4 - Quadratic Equations.

(Notes)

[Topic-1] Solution of Quadratic Equation.

(1) A quadratic equation in the variable x is of the form $ax^2 + bx + c = 0$, where a, b and c are real numbers $a \neq 0$.

(2) The values of x that satisfies an equation is called the solution or roots or zeros of the equation.

(3) A real number α is said to be a solution / root or zero of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

(4) A quadratic equation can be solved by the following algebraic methods:

(i) Splitting the middle term
(ii) Making perfect square (Completing square method)
(iii) Using quadratic formula.

(5) If $ax^2 + bx + c = 0$, $a \neq 0$. can be reduced to the product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.

(6) Method for splitting the middle term of the equation $ax^2 + bx + c = 0$, where $a \neq 0$.

(i) Form the product "ac".
(ii) Find the pair of numbers b_1 and b_2 whose product is "ac" and whose sum is "b" (if you can't find such number, it can't be factorised.
(iii) Split the middle term using b_1 and b_2, that expresses the term bx as $b_1x + b_2x$. Now factor by grouping the pairs of terms.

(7) Roots of the quadratic equation can be found by equating each linear factor to zero. Since product of two numbers is zero, if either or both of them are zero.

(8) Any quadratic equation can be converted in to the form $(x+a)^2 - b^2 = 0$ by adding and subtracting some terms. This method of finding the roots of quadratic equation is called the method of making the perfect the square.

9) Method of making the perfect square for quadratic equation $ax^2+bx+c=0$, $a \neq 0$.

(i) Dividing throughout by a, we get $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

(ii) Multiplying and dividing the coefficient of x by 2

$$x^2 + 2\frac{b}{2a} + \frac{c}{a} = 0$$

(iii) Adding and subtracting $\frac{b^2}{4a^2}$, $x^2 + 2\cdot\frac{b}{2a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} = 0$

$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}, \quad x+\frac{b}{2a} = \frac{\pm\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}, \therefore x = \frac{-b+\sqrt{b^2-4ac}}{2a}, x = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

(10) Quadratic Identities:

(i) $(a+b)^2 = a^2 + 2ab + b^2$

(ii) $(a-b)^2 = a^2 - 2ab + b^2$

(iii) $a^2 - b^2 = (a+b)(a-b)$

(11) For quadratic eq. $ax^2+bx+c=0$, the expression b^2-4ac is known as discriminant. (i) If $b^2-4ac > 0$, eq. has two distinct real roots.

(ii) If $b^2-4ac = 0$, eq. has two equal real roots

(iii) If $b^2-4ac < 0$, eq has no real roots.