Chapter 10 = Light - Reflection and Refraction

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Physics (Science) - Class X

Chapter 10 : Light - Reflection and Refraction.

Topic-1 : Reflection of light, Image formed by spherical mirrors.

Introduction

Light is a form of energy. Light is needed to see things around us. An object reflects light that falls on it. This reflected light, when received by our eyes, enables us to see things.

Luminous Objects : The objects like the sun, other stars, electric bulb, tube-light, torch, candle and fire etc, which emit their own light are called luminous objects. We can see the luminous objects due to the light emitted by them.

Non-Luminous objects : Those objects which do not emit light themselves but only reflect (or scatter) the light which falls on them, are called non-luminous objects. A flower, chair, table, book, trees, fan, bed, mirror etc are all non-luminous objects. We can see the non-luminous objects because they reflect light (received from a luminous object) into our eyes.

Light travels in straight line.

According to wave theory : light consists of electromagnetic waves which do not require a material medium (like solid, liquid or gas) for their propagation.
The wavelength of visible light is very small (4×10^{-7} to 8×10^{-7} m).
Speed of light wave is very high (3×10^{8} m/s in vacuum).

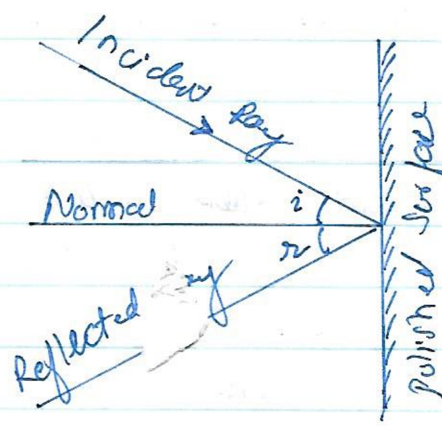
According to particle theory : Light is composed of particles which travel in a straight line at very high speed. The elementary particle that defines light is the 'photon'.

10.1 Reflection of Light

A highly polished surface, such as a mirror, reflects most of the light falling on it.

Laws of reflection light:

(i) The angle of incidence is equal to the angle of reflection, and



(ii) The incident ray, the normal to the mirror at the point of incidence and the reflected ray, all lie in the same plane.

These laws of reflection are applicable to all types of reflecting surfaces including spherical surfaces.

Silver metal is one of the best reflectors of light. Ordinary mirrors are made by depositing a thin layer of silver metal on the back side of a plane glass sheet. The silver layer is then protected by a coat of red paint.

Image formed by a plane mirror

(i) Image formed by a plane mirror is always virtual and erect.

(ii) The size of the image is equal to that of the object.

(iii) The image formed is as far behind the mirror as the object is in front of it.

(iv) The image is laterally inverted.

10.2 Spherical Mirrors:

Mirrors whose reflecting surfaces are spherical, are called spherical mirrors. The reflecting surface of a spherical mirror, may be curved inwards or outwards.

Concave mirror: A spherical, whose reflecting surface is curved inwards, that is, faces towards the centre of the sphere, is called a concave mirror.

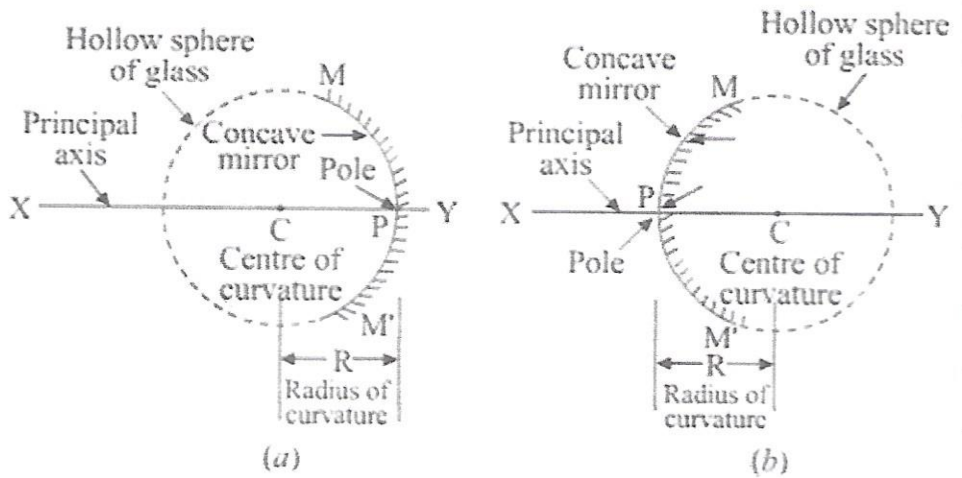
Convex mirror: A sphere whose reflecting surface is curved outwards, is called a convex mirror.

The spoon curved inwards can be approximated to a concave mirror and the surface of the spoon bulged outwards can be approximated to a convex mirror.

Pole (P): The centre of the reflecting surface of a spherical mirror is a point called the pole. It lies on the surface of the mirror.

Centre of Curvature (C): The centre of sphere which form the reflecting surface of spherical mirror is called center of curvature.

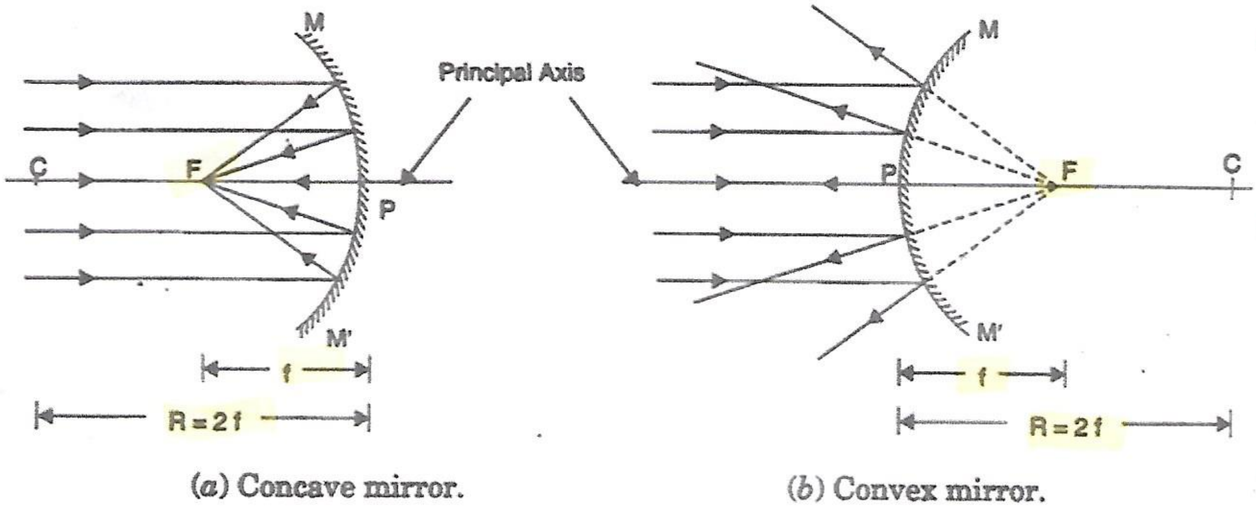
Radius of Curvature (R): The radius of the sphere of which the reflecting surfaces of a spherical mirror forms a part, is called the radius of curvature of the mirror.



Diagrams to show how a concave mirror and a convex mirror can be considered to be the part of a hollow sphere of glass.

Principal Axis : A straight line passing through the pole and the centre of curvature of a spherical mirror is called the principal axis.

When light from the Sun is converged at a point on a concave mirror, a sharp, bright spot is formed. This spot of light is the image of the Sun on the sheet of paper. This point is the focus of the concave mirror. The heat produced due to the concentration of sunlight ignites the paper.



(a) Concave mirror. (b) Convex mirror.

Spherical mirrors.

Principal focus (F) :
A number of rays parallel to the principal axis are falling on a concave mirror / convex mirror. They are all meeting / intersecting at a point on the principal axis of the mirror. This point is called the principal focus of the concave mirror. It is represented by F

Focal length (f) : The distance between the pole and the principal focus of a spherical mirror is called the focal length.

Aperture : The diameter of the reflecting surface of spherical mirror is called its aperture. Distance MM' represent the aperture.

Radius of curvature (R) = 2 × focal length (f)

$$R = 2f$$

Principal focus (F) of a spherical mirror lies midway between the pole and centre of curvature.

10.2.1 Image formed by Spherical Mirrors:

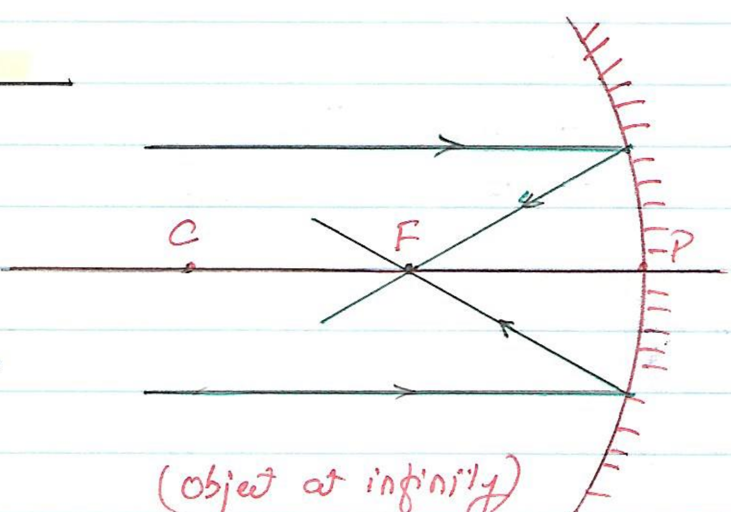
Image formation by Concave Mirror :

Formation of image depends upon the position of the object. There are six possibilities of the position of the object in the case of concave mirror.

a) Object at infinity
b) Object between infinity and center of curvature (C)
c) Object at centre of curvature (C)
d) Object between centre of curvature (C) and Principal focus (F).
e) Object at Principal focus (F)
f) Object between Principal focus (F) and Pole (P)

(a) Object at infinity:

Properties of image :
- Point sized
- Highly diminished
- Real and inverted.

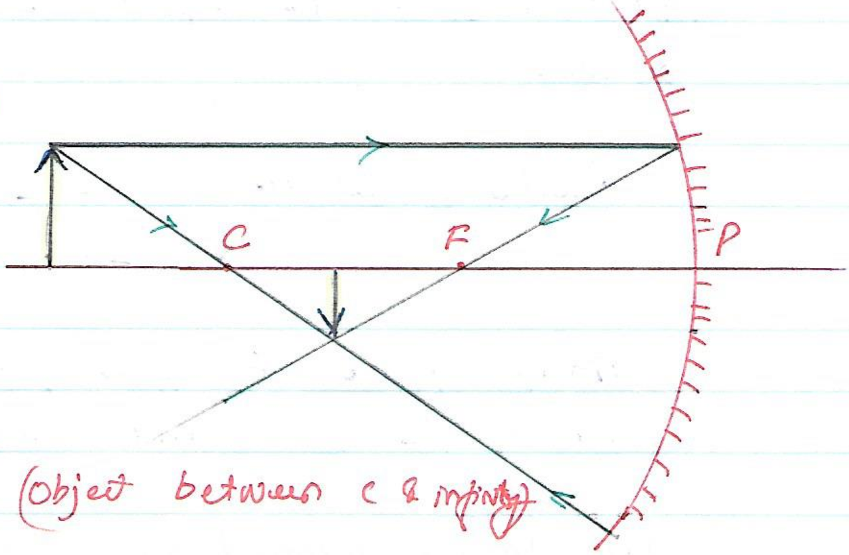


(object at infinity)

(b) Object between infinity and center of curvature

Properties of image:

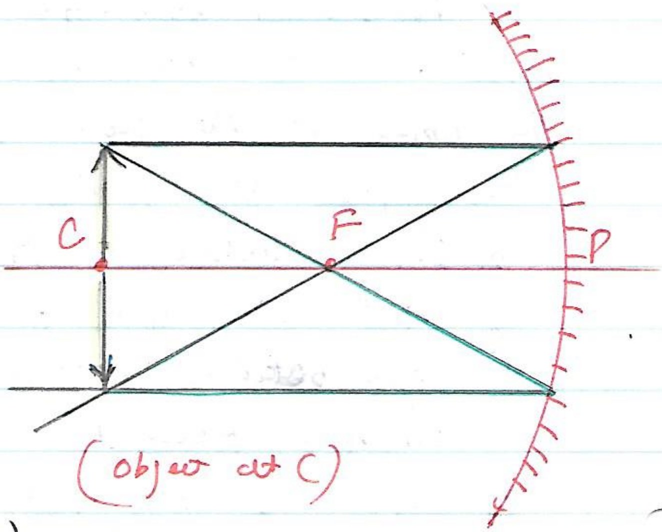
- Diminished compared to the object
- Real and inverted.



c) Object at Centre of Curvature (C)

Properties:

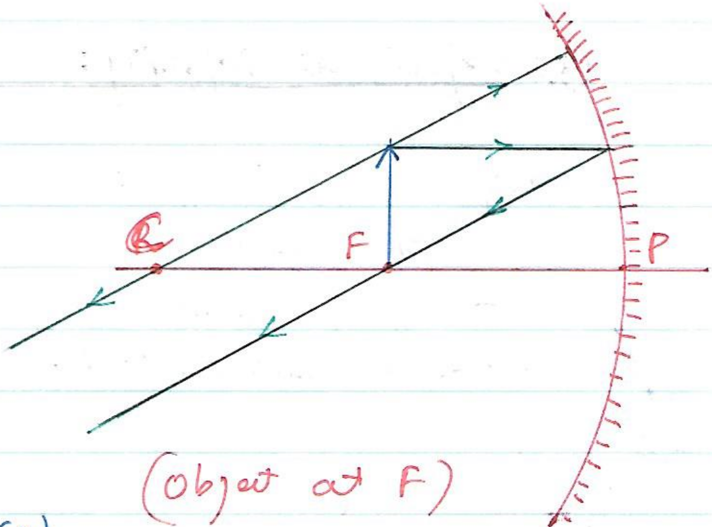
- Same size of the object
- Real and inverted.



d) Object at Principal Focus (F)

Properties:

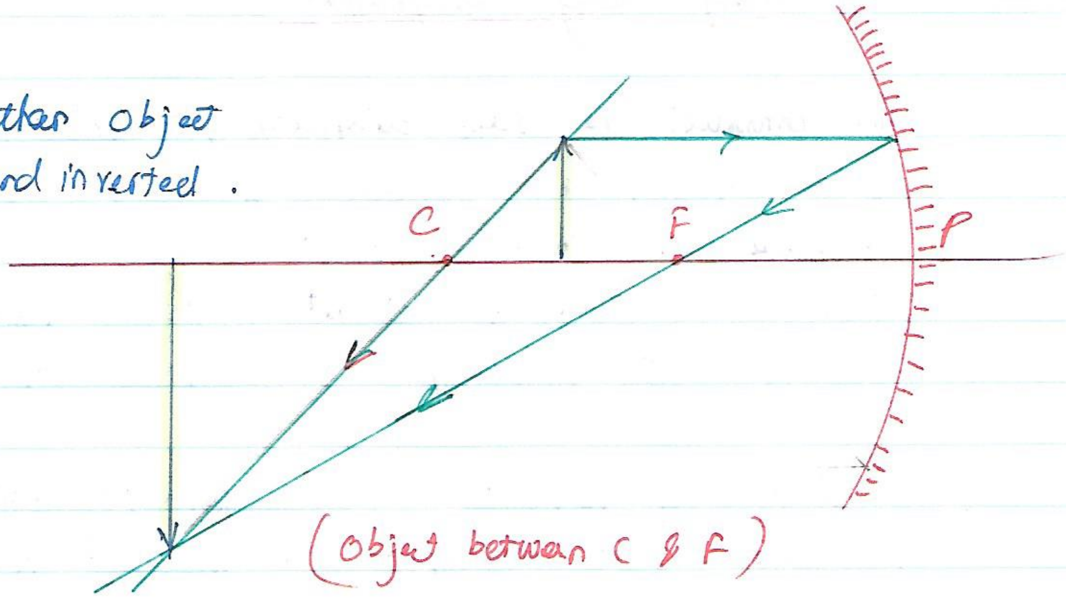
- Highly Enlarged
- Real and inverted.



(e) Object between principal focus (F) and centre of curvature (C)

Properties:

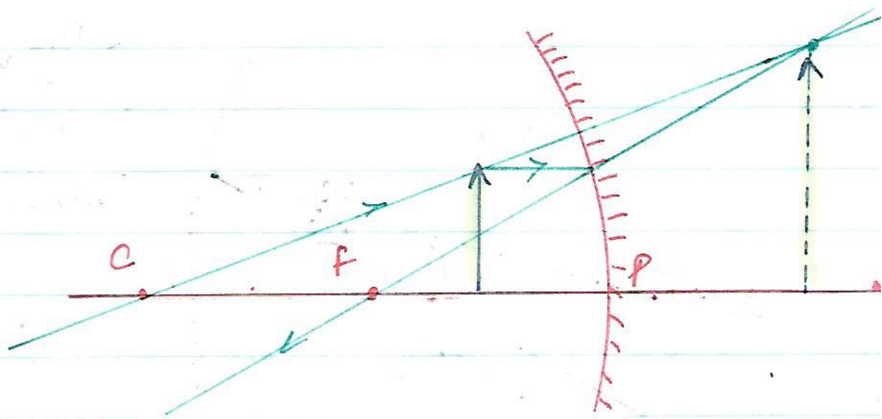
- Larger than object
- Real and inverted.



(f) Object between Principal focus (F) and Pole (P)

Properties:

- Enlarged
- Virtual and erect.



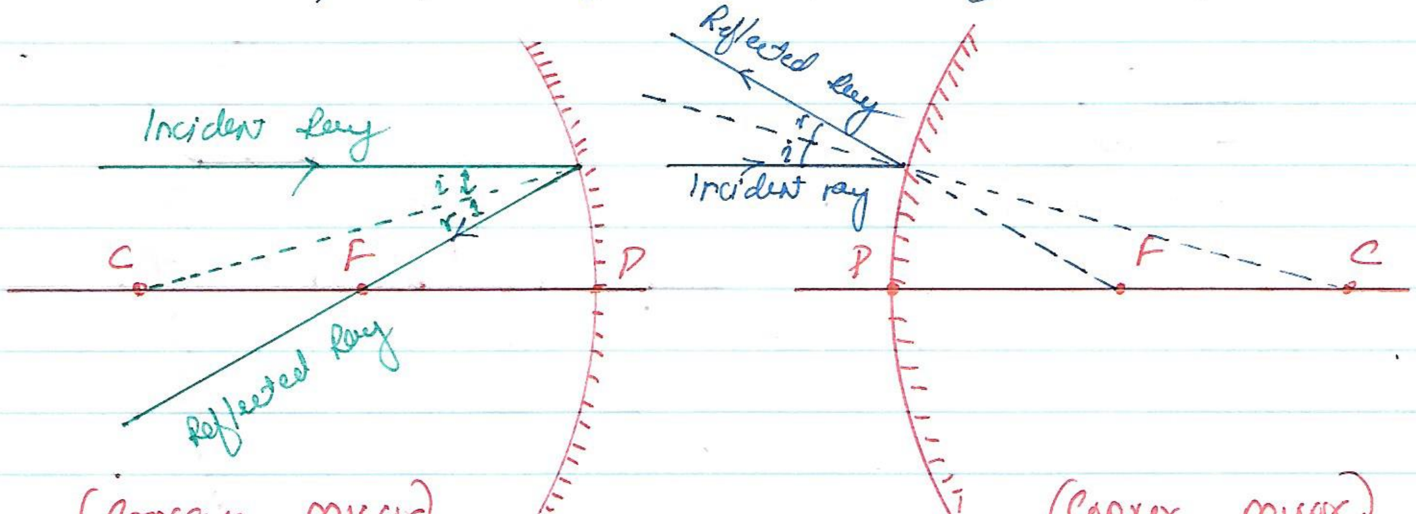
Summary of the Images Formed by a Concave and Convex Mirror

Position of object	Position of Image	Size of Image	Nature of Image
Concave Mirror			
At infinity	At Focus F	Highly diminished	Real and inverted
Beyond C	Between F and C	Diminished	Real and inverted
At C	At C	Same size	Real and inverted
Between F and C	Beyond C	Enlarged	Real and inverted
At F	At infinity	Highly enlarged	Real and inverted
Between P and F	Behind the mirror	Enlarged	Virtual and erect
Convex Mirror			
At infinity	At focus	Highly diminished	Virtual point size
Anywhere on Principal axis	Between pole & focus	Diminished	Virtual and erect

10.2.2 Representation of Images formed by Spherical Mirrors using Ray Diagram.

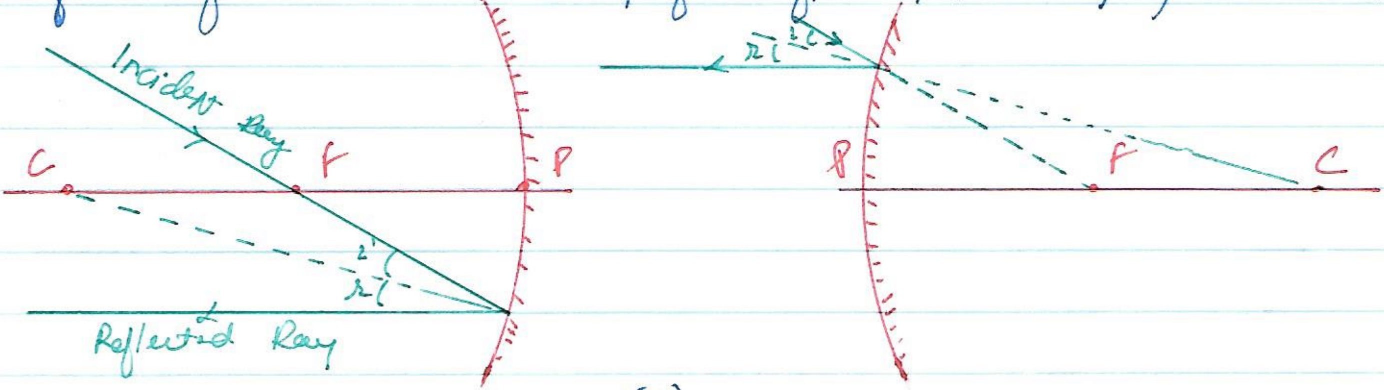
To construct the ray diagram, in order to locate the image of an object, an arbitrarily large number of rays emanating from a point could be considered.
The intersection of atleast two reflected rays give the position of image of the point object.

(i) A ray parallel to the principal axis, after reflection, will pass through the principal focus in case of a concave mirror or appear to diverge from the principal focus in case of a convex mirror

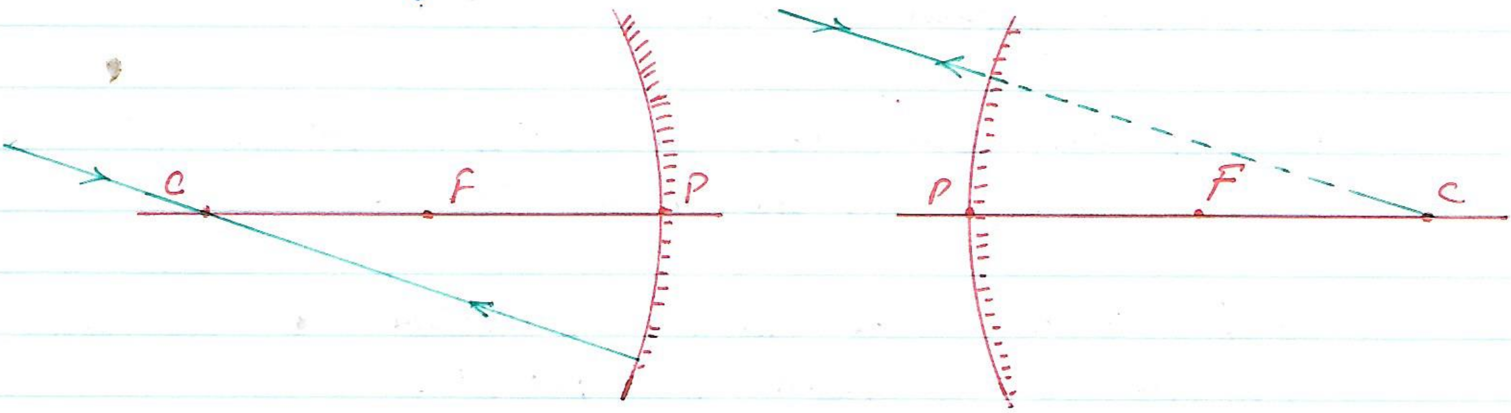


(Concave mirror) (Convex mirror)

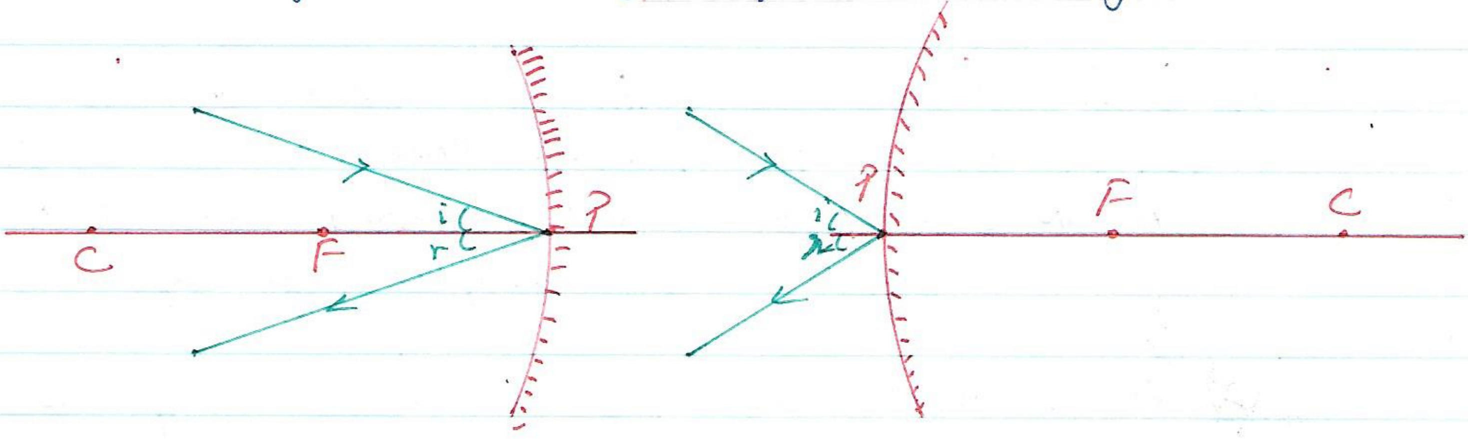
(ii) A ray passing through the principal focus of a concave mirror or a ray which is directed towards the principal focus of a convex mirror, after reflection, will emerge parallel to axis.



(iii) A ray passing through the centre of curvature of a concave mirror or directed in the direction of the centre of curvature of a convex mirror, after reflection, is reflected back along the same path.



(iv) A ray incident obliquely to the principal axis, towards a point P (pole of the mirror), on the concave mirror or a convex mirror, is reflected obliquely. But laws of reflection are obeyed.



Remember that in all the above cases the laws of reflection are followed. At the point of incidence, the incident ray is reflected in such a way that the angle of reflection equals the angle of incidence.

Use of concave mirrors

(1) Concave mirrors are commonly used in torches, search-lights and vehicles headlights to get powerful parallel beams of light.

(2) They are often used as shaving mirrors to see a larger image of the face.

(3) The dentists use concave mirrors to see large images of the teeth of patients.

(4) Large concave mirrors are used to concentrate sunlight to produce heat in solar furnaces.

Uses of convex mirrors

(1) Convex mirrors are commonly used as rear-view (wing) mirrors in vehicles. These mirrors are fitted on the sides of the vehicle, enabling the driver to see traffic behind him/her for safe driving.

(2) Convex mirrors are preferred because they always give an erect, though diminished.

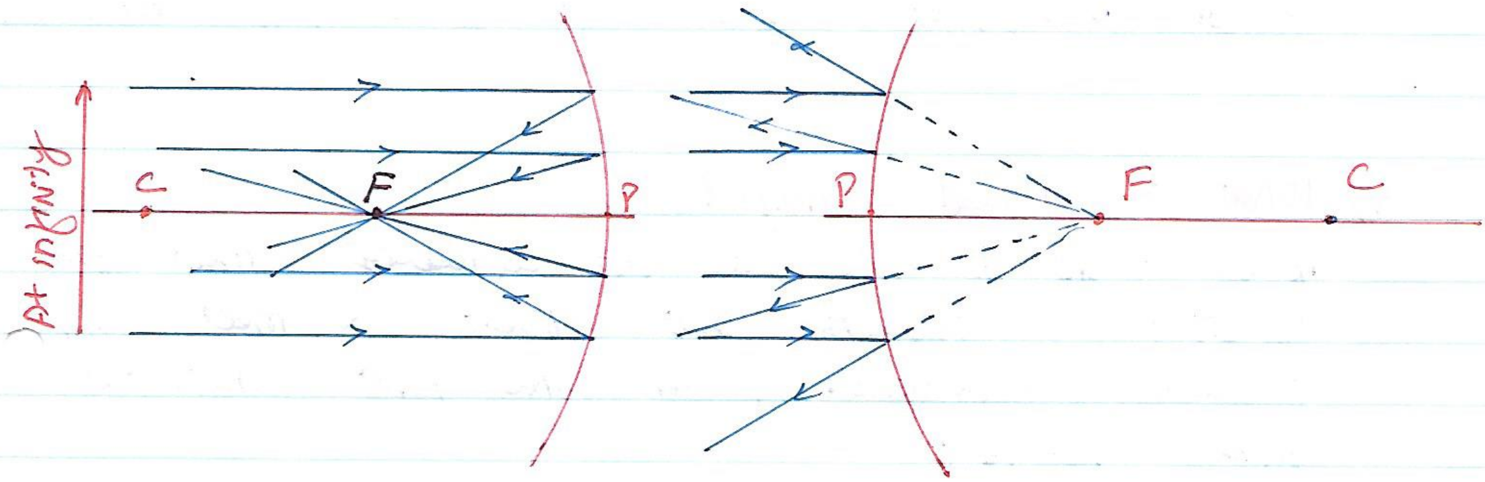
(3) They have wider field of view as they are curved outwards.

(4) Thus, convex mirrors enable the driver to view much larger area than would be possible with a plane mirror.

Question & Answers

1) Define the principal focus of a concave mirror.

ANS. Light rays that are parallel to the principal axis of a concave mirror converge at a specific point on its principal axis after reflecting from the mirror. This point is known as the principal focus of the concave mirror.



2) Radius of curvature of a spherical mirror is 20 cm. What is its focal length?

ANS. Radius of curvature $R = 20\,cm$

$R = 2 \times$ focal length (f)

$R = 2f$

$f = \frac{R}{2} = \frac{20}{2} = 10\,cm$ ANS.

3) Name the mirror that can give an erect and enlarged image of an object.

ANS. When an object is placed between the pole (P) and principal focus (f) of a concave mirror, the image formed is virtual, erect and enlarged.

4) Why do we prefer a convex mirror as a rear-view mirror in vehicles?

ANS) Convex mirrors give a virtual, erect, and diminished image of the objects placed in front of it. They are preffered as a rear view mirror in vehicles because they give a wider field of view, which allows the driver to see most of the traffic behind him.

5) What is virtual image?

If the rays of light do not actually meet after reflection or refraction, but appear to meet when produced backwards, then that point constitutes virtual image.

6.) What is the range of wavelengths of the visible light?

ANS) Wavelength of visible light $= 4 \times 10^{-7}$ m to 8×10^{-7} m.

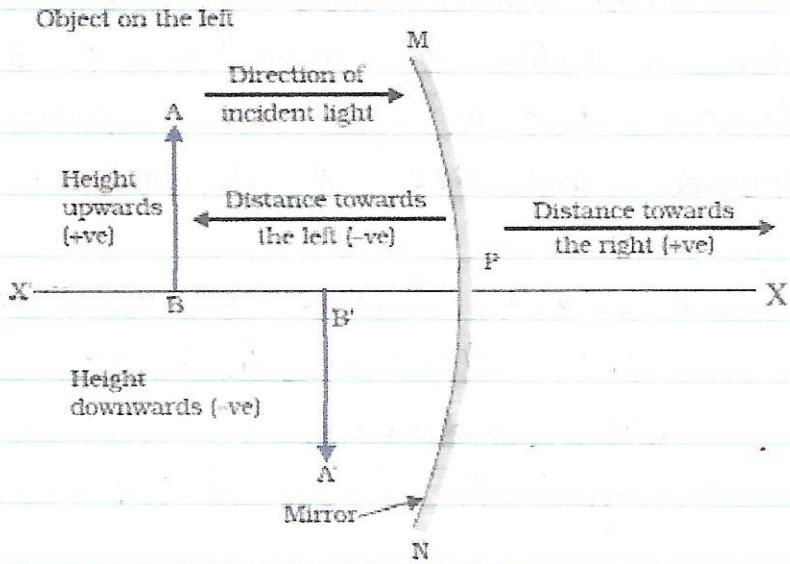
7.) What is the minimum distance between an object and its real image in case of a concave mirror?

ANS) Zero (0). When the object is at C.

8) Explain why a ray of light passing through the centre of curvature of a concave mirror, gets reflected along the same path.

Ans. The ray passing through the centre of curvature is incident to the mirror along its normal so $\angle i = \angle r = 0$. Therefore, the ray retrace its path.

10.2.3 Sign Convention for Reflection by Spherical Mirrors



10.2.4 Mirror formula and Magnification

Mirror Formula ⇒ $\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$

v - distance of the image from the pole of the mirror
u - distance of the object from its pole
f - focal length.

This formula is valid in all situations for all spherical mirrors for all positions of the object. Use Cartesian sign convention while substituting numerical values for u, v, f and R in the mirror formula.

Magnification

$$m = \frac{\text{Height of the image } (h')}{\text{Height of the object } (h)}$$

$$m = \frac{h'}{h} = -\frac{v}{u}.$$

A negative sign indicates real image
A positive sign indicates virtual image.

Numericals

1) A convex mirror used for rear-view on an automobile has a radius of curvature of 3 m. If a bus is located at 5 m from this mirror, find the position, nature and size of the image.

Ans

Radius of curvature $(R) = +3$ m

Object distance $(u) = -5$ m

Image distance $(v) = ?$

Height of image $h' = ?$

Focal length $f = \frac{R}{2} = \frac{3}{2}$ m $= 1.5$ m

Since, using mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{1.5} - \frac{1}{-5} = \frac{1}{1.5} + \frac{1}{5}$$

$$\frac{1}{v} = \frac{5 + 1.5}{7.5} = \frac{6.5}{7.5}$$

$$v = \frac{7.5}{6.5} = +1.15 \text{ m} \quad (\text{Image distance})$$

The image is 1.15m at the back of the mirror.

$$\text{Magnification, } m = \frac{h'}{h} = -\frac{v}{u} = -\frac{1.15}{5} = +0.23$$

The image is virtual, erect and smaller in size by a factor of 0.23

2) An object, 4 cm in size, is placed at 25 cm in front of a concave mirror of focal length 15 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Find the nature and the size of the image.

Solution: Object - size $h = +4$ cm
Object - distance, $u = -25$ cm
Focal length, $f = -15$ cm
Image distance $v = ?$
Image size $h' = ?$

Using mirror equation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-15} - \frac{1}{-25} = -\frac{1}{15} + \frac{1}{25}$$

$$\frac{1}{v} = \frac{-5+3}{75} = \frac{-2}{75}$$

$$v = -\frac{75}{2} = -37.5 \text{ cm}$$

The screen should be placed at 37.5 cm from the mirror.
The image is real.

Magnification $m = \frac{h'}{h} = -\frac{v}{u} \Rightarrow h' = -\frac{v}{u} \times h$

$$h' = -\frac{(-37.5) \times (4)}{-25} = -6 \text{ cm}$$

$\therefore h' = -6$ cm

The image is inverted and enlarged.

3) Find the focal length of a convex mirror whose radius of curvature is 32 cm.

Ans: Radius of curvature $R = 32$ cm

$R = 2 \times$ focal length (f)

$R = 2f$

$$f = \frac{R}{2} = \frac{32}{2} = 16 \text{ cm} \quad \text{Ans.}$$

4) A concave mirror produces three times magnified (enlarged) real image of an object placed at 10 cm in front of it. Where is the image located?

Ans:

Magnification $m = -3$

Object distance $u = -10$ cm

$$m = \frac{h'}{h} = -\frac{v}{u} \Rightarrow -\frac{v}{u} = -3$$

$$v = 3u = 3 \times (-10 \text{ cm})$$

$$v = -30 \text{ cm}$$

Negative sign indicates that an inverted image is formed at a distance of 30 cm in front of the concave mirror.

Topic - 2 : Refraction , Lenses , Power of Lens .

10·3 Refraction light

The phenomenon of change in the path of light from one medium to another is called refraction of light. The cause of refraction is the change in the speed of light as it goes from one medium to another medium. Larger the difference in speed of light between the two media across the interface, the greater will be the angle of bending and vice·versa.

Real life examples :

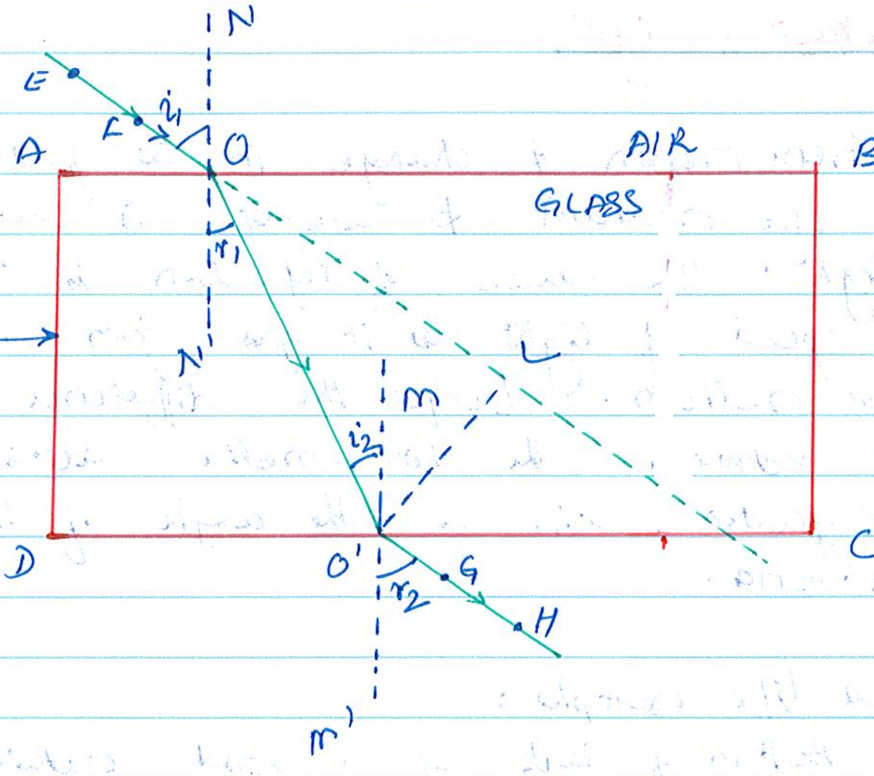
(i) Bottom of tank or a pond containing water appears to be raised.

(ii) When a thick glass slab is placed over some printed matter, the letters appear raised when viewed through the glass slab.

(iii) Pencil partly immersed in water in a glass tumbler appears to be displaced at the interface of air and water.

(iv) Lemon kept in water in a glass tumbler appears to be bigger than its actual size, when viewed from the sides.

10.3.1 Refraction through a Rectangular Glass Slab.



When ray of light enters from a rarer medium into a denser medium, it bends towards normal to the point of incidence. On the contary, when ray of light enters into a rarer medium from a denser medium it bends away from the normal.

Ray emerging after the denser medium goes in the same direction and parallel to the incident ray.

The angle between the incident ray and normal is called angle of incidence and it is denoted as 'i'. The angle between refracted ray and normal is called the angle of refraction denoted by r.

Laws of refraction of light :

(i) The incident ray, the refracted ray and the normal to the interface of two transparent media at the point of incidence, all lie in the same plane.

(ii) The ratio of sine of angle of incidence to the sine of angle of refraction is a constant, for the light of a given colour and for the given pair of media. The law is also known as Snell's law of refraction. That is, $\frac{\sin(i)}{\sin(r)}$ = constant.

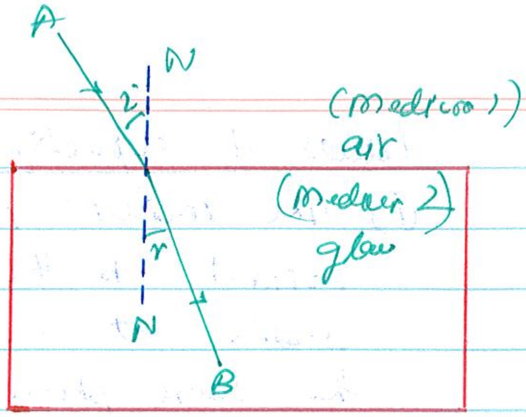
This constant value is called refractive index of the second medium with respect to the first.

10.3.2 The Refractive Index

A ray of light changes its direction when it enters from one medium to another medium. This happens because speed of light is different in different media. For example, the speed of light is 3×10^8 m/s (2.99×10^8 m/s) in vacuum, and it is 2.98×10^8 m/s in air.

Refractive index is the extent of change of direction of light in given pair of media. The refractive index is a relative value of speed of light in the given pair of media.

Consider a ray of light travelling from medium 1 into medium 2.



v_1 — speed of light in medium 1
v_2 - speed of light in medium 2

The refractive index of medium 2 with respect to medium 1 is given by the ratio of speed of light in medium 1 and the speed of light in medium 2. This is represented by n_{21}.

$$n_{21} = \frac{v_1}{v_2}$$

Similarly $$n_{12} = \frac{v_2}{v_1}$$

Absolute Refractive Index:

When one medium is taken as vacuum and speed of light is taken in it, then the refractive index of second medium with respect to vacuum is called Absolute Refractive Index and is denoted by n_2

$$n_2 = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in given medium.}}$$

If c is the speed of light in air and v is the speed of light in the medium, then refractive index of medium

$$n_m = \frac{\text{Speed of light in air}}{\text{Speed of light in medium}} = \frac{c}{v}$$

The absolute refractive index of a medium is simply called its refractive index.

Since, Refractive Index is the relative value of the speed of light of a medium with respect to the speed of light in vacuum, then light will travel faster in medium having lower lower value of refractive index.

Optical Density :

Medium having greater value of refractive index is called optically denser medium, this mean light will travel at slower speed in optically denser medium compared to in an optically rarer medium.

Q) A ray of light travelling in air enters obliquely into water. Does the light ray bend towards the normal or away from the normal? Why?

ANS) The light ray bends towards the normal. When a ray of light travels from an optically rarer medium to an optically denser medium its speed slows down and it bends towards the normal. Since water is optically denser than air, a ray of light travelling from air into the water will bend towards the normal.

Q) Light enters from air to glass having refractive index 1.5. What is the speed of light in the glass? The speed of light in vaccum is 3×10^8 m/s.

Ans) $n_m = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}} = \frac{c}{v}$

Speed of light in vacuum $c = 3 \times 10^8$ m/s
Refractive index of glass $n_m = 1.5$

$$1.5 = \frac{3 \times 10^8}{v}$$

$$v = \frac{3 \times 10^8}{1.5}$$

$v = 2 \times 10^8$ m/s (Speed of light in glass)

Q) The absolute refractive indices of glass and water are 4/3 and 3/2 respectively. If the speed of light in glass is 2×10^8 m/s, calculate the speed of light in (i) ~~vacuum~~ air (ii) water.

Ans) Refractive index of Glass $= \frac{\text{Speed of light in ~~vacuum~~ air}}{\text{Speed of light in glass}}$

$$n_{21} = \frac{v_1}{v_2}$$

$$\frac{4}{3} = \frac{v_1}{2 \times 10^8} \Rightarrow v_1 = \frac{4 \times 2 \times 10^8}{3} =$$

$v_1 = 2.67 \times 10^8$ m/s (Speed of light in air)

Refractive index of water $= \frac{\text{Speed of light in vacuum}}{\text{Speed of light in water}}$

$$n_{12} = \frac{u_2}{u_1}$$

$$\frac{3}{2} = \frac{3\times10^8 \text{ (speed of light in vacuum)}}{u_1 \text{(speed of light in water)}}$$

$$u_1 = \frac{3\times10^8 \times 2}{3} = 2\times10^8 \text{ m/s}$$

(8) The refractive indices of glass and water with respect to air are 3/2 and 4/3 respectively. If speed of light in glass is 2×10^8 m/s, find the speed of light in water.

ans) Refractive index of a medium

$$= \frac{\text{speed of light in air}}{\text{speed of light in medium}}$$

$$\frac{3}{2} = \frac{\text{speed of light in air}}{2\times10^8 \text{ m/s}}$$

$\therefore$ speed of light in air $= \frac{3}{2}\times10^8\times2$ m/s

$= 3\times10^8$ m/s

Refractive index of medium $= \frac{\text{speed of light in air}}{\text{speed of light in water}}$

$$\frac{4}{3} = \frac{3\times10^8}{\text{speed of light in water}}$$

Speed of light in water $= \frac{3}{4}\times3\times10^8$ m/s.

$= 2.25\times10^8$ m/s.

Q) Find out from Table (10.3), the medium having highest optical density. Also find the medium with lowest optical density.

Ans) Highest optical density = Diamond
Lowest optical density = Air

Optical density of a medium is directly related with the refractive index of that medium. A medium which has the highest refractive index will have the highest optical density, and vice versa.

Q.) You are given kerosene, turpentine and water. In which of these does the light travel fastest?

Ans)

Refractive Index of medium $n_m = \dfrac{\text{Speed of light in air } (c)}{\text{Speed of light in medium } (v)}$

$$n_m = \frac{c}{v}$$

$$v = \frac{c}{n_m} \Rightarrow v \propto \frac{1}{n_m} \quad [\because c \text{ is constant}]$$

It can be inferred from the relation that light will travel the slowest in the material which has the highest refractive index and travel the fastest in the material which has the lowest refractive index.

Refractive index of Kerosene = 1.44
Turpentine = 1.47
water = 1.33.

$\therefore$ Light travels the fastest in water.

Q) The refractive index of diamond is 2.42. What is the meaning of this statement?

Ans)

Refractive index of the medium

$$n_m = \frac{\text{speed of light in air (c)}}{\text{speed of light in the medium (v)}}$$

The refractive index of diamond is 2.42. This suggest that the speed of light in diamond will be reduced by a factor of 2.42 as compared to its speed in air.

Q.) "A ray of light incident on a rectangular glass slab immersed in any medium emerges parallel to itself." Draw a diagram to justify the statement.

Q.) State two laws of refraction.

Q.) State a condition for no refraction of light entering from one medium to another.

Ans) (i) Light incident normally

(ii) Equal refractive index of two media.

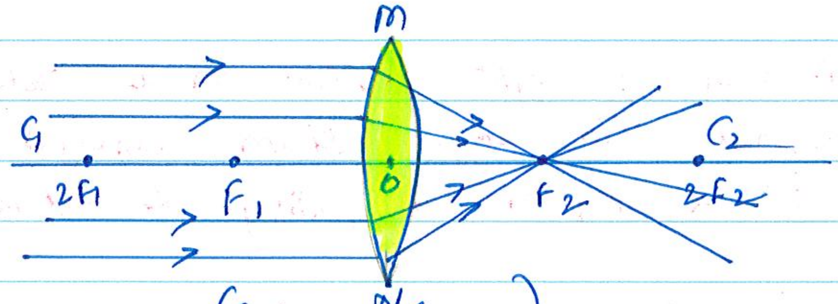
Q.) Why is the refractive index of atmosphere different at different altitudes?

Ans) Because the air density change with altitude.

10.3.3 Refraction by Spherical lenses.

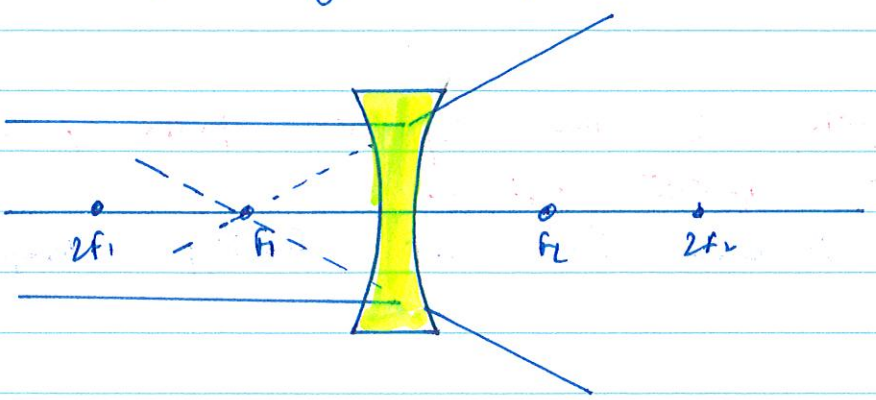
Lens : A transparent material bound by two surfaces, of which one or both surfaces are spherical, forms a lens.

Convex lens : A lens having two spherical surface bulging outwards is called convex lens. It is also known as a double convex lens. Convex lens converge light rays, hence called converging lenses.



(Convex lense)

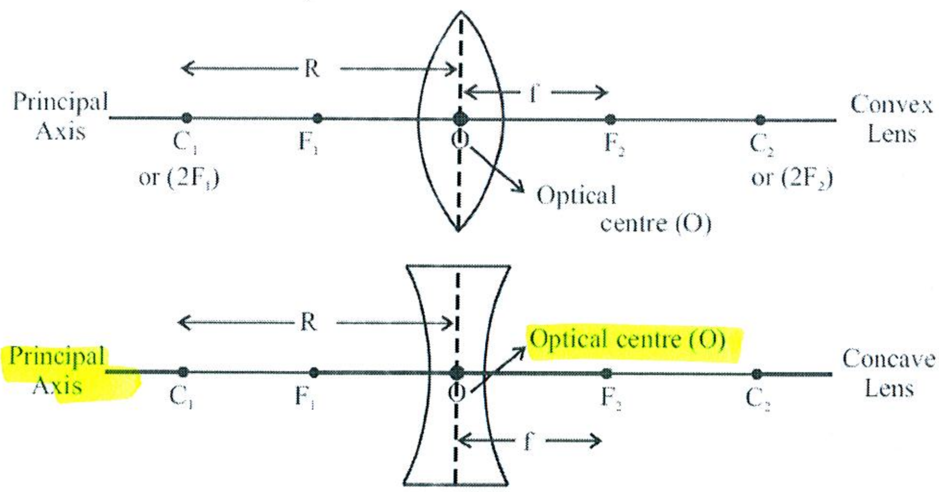
Concave lense : A lens having two spherical surface bulging inwards is called concave lens. It is thicker at the edges than at the middle. Such lenses diverge light rays, hence called diverging lenses.



Centre of curvature: The centre of sphere of part of which a lens is formed is called the centre of curvature. We have two centres of curvature C_1 & C_2.

Focus: Point at which parallel rays of light converge in a concave lens and parallel rays of light diverge from the point is called focus or Principal focus of lens.

Few Basic Terms related to spherical lens.



Principal Axis: Imaginary line that passes through the center of curvature of lens.

Optical Centre: The central point of a lens is called its optical centre. A ray passes through optical centre of a lens without any deviation.

Radius of Curvature: The distance between optical centre and centre of curvature is called the radius of curvature, denoted by R.

focal length : The distance between optical center and principal focus is called focal length of lens.

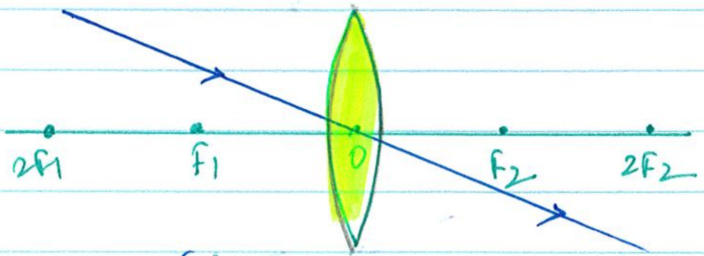
$R = 2f$ or $f = \frac{R}{2}$

Q.) What happens when parallel rays of light are incident on a lens?

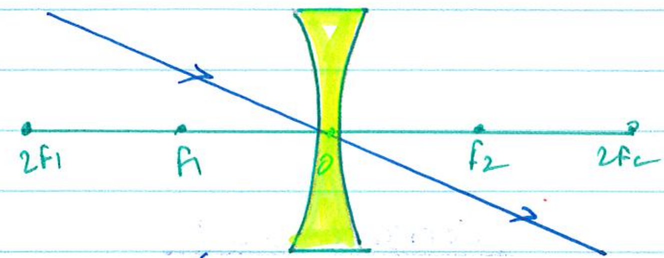
Ans) The paper begins to burn producing smoke. It may even catch fire after a while. The light from sun constitute parallel rays of light. These rays were converged by the lens at the sharp bright spot formed on the paper. Bright spot you got on the paper is a real image of the sun. The concentration of the sunlight at a point generated heat. This caused the paper to burn.

Rules for the formation of Image by the lenses:

(1) In both the lenses a ray of light passing through the optical center goes without any deviation.



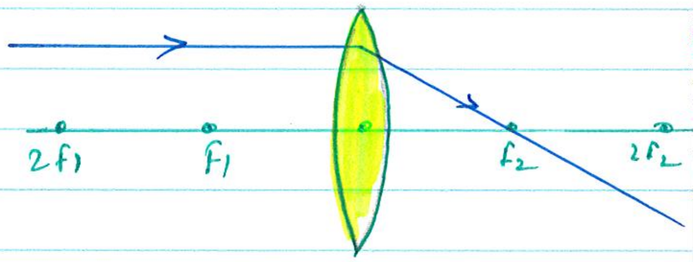
(Convex lens)



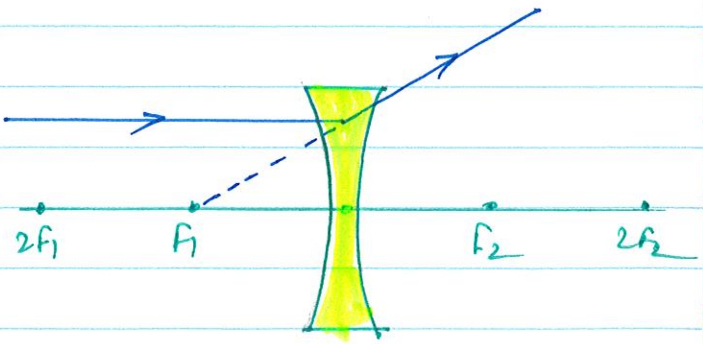
(Concave lens)

(2) A ray of light parallel to principal axis:

<u>Convex lens:</u> After refraction passes through the focus on other side of lens.



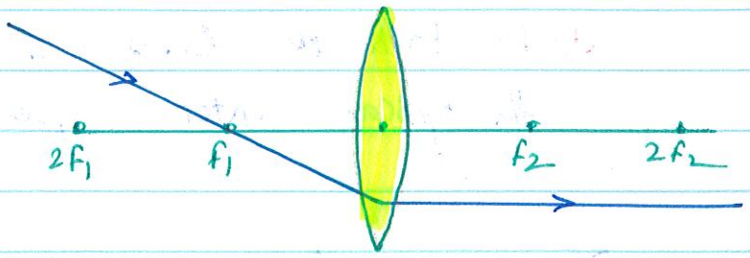
<u>Concave lens:</u> A ray of light appears to diverge from the focus on the same side of lens.



(3). A ray of light passing through focus :

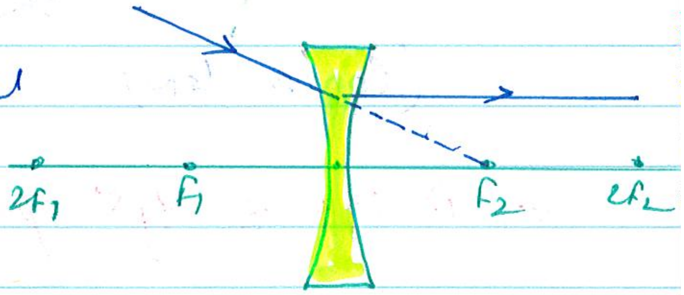
Convex lens :

after refraction goes parallel to the principal axis



Concave lens :

After refraction goes parallel to the principal axis.



Summary

Rules for the Formation of Image by the Lenses.

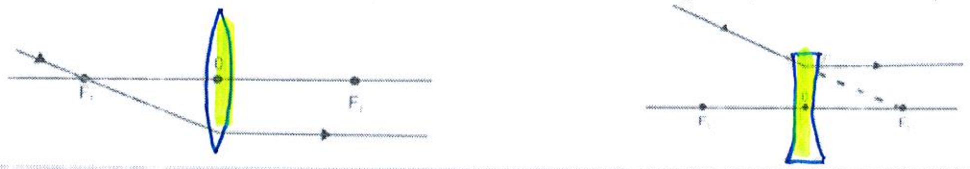
1) In both the lenses a ray of light passing through the optical center goes without any deviation.

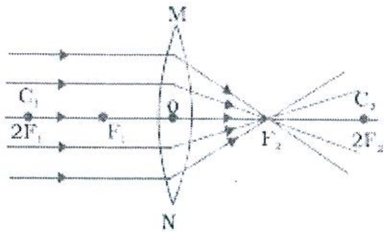
A ray of light parallel to the principal axis after refraction passes through the focus on the other side of the lens.

A ray of light appears to diverge from the focus on the same side of the lens.

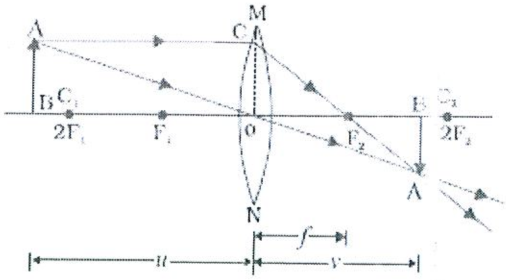
A ray of light passing through the focus after refraction goes parallel to the principal axis.

A ray of light directed towards the focus after refraction goes parallel to the principal axis.

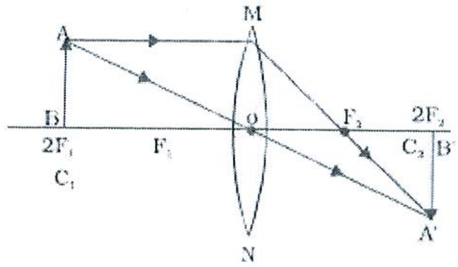




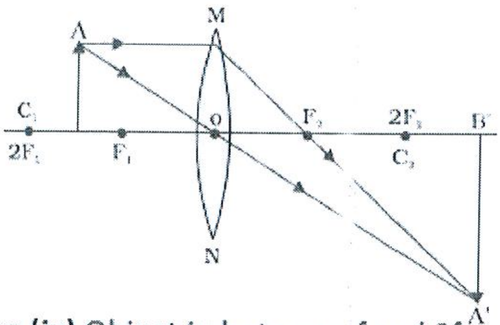
Case (i) Object at infinity



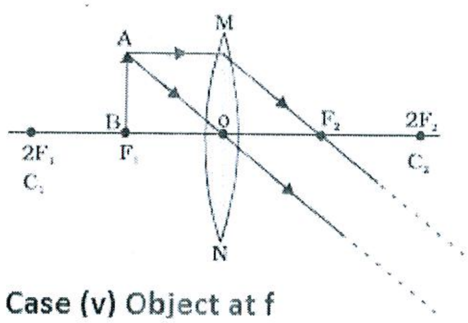
Case (ii) Object at beyond 2f



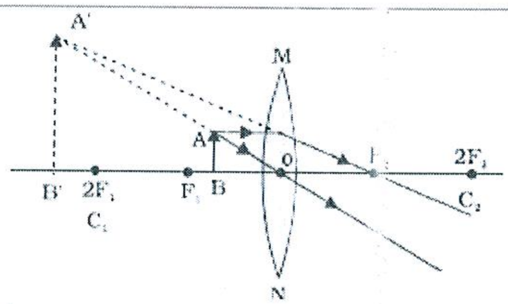
Case (iii) Object at 2f



Case (iv) Object in between f and 2f



Case (v) Object at f



Case (vi) Object distance < f

Object position	Image position	Size of image	Nature of image
At infinity	At F_2	Extremely small	Real and inverted
Behind $2F_1$	Between F_2 and $2F_2$	Small	Real and inverted
At $2F_1$	At $2F_2$	Same as that of the object	Real and inverted
Between $2F_1$ and F_1	Beyond $2F_2$	Enlarged	Real and inverted
At F_1	At infinity	Highly enlarged	Real and inverted
Between F_1 and O	Same side of the lens	Enlarged	Virtual and erect

10.3.7 Lens formula and Magnification

u - object distance
v - image distance
f - focal length

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

The lens formula given above is general and is valid in all situations for any spherical lens. Take proper care of the signs of different quantities.

Magnification :

$$m = \frac{\text{Height of Image}}{\text{Height of object}} = \frac{h'}{h} = \frac{v}{u}$$

10.3.8 Power of a lens.

The degree of convergence or divergence of light rays achieved by a lens is expressed in terms of its power. The power of a lens is defined as the reciprocal of its focal length, represented by letter P

$$P = \frac{1}{f}$$

The SI unit of power of a lens is 'dioptre' denoted by D. If f is expressed in metres then power is expressed in dioptre $1 D = 1 m^{-1}$

Power of a convex lens is positive and that of a concave lens is negative.

Numerical Problem on Lens.

Q.) A concave lens has focal length of 15cm. At what distance should the object from the lens be placed so that it forms an image at 10cm from the lens? Also, find the magnification produced by the lens.

Solution:

A concave lens always forms a virtual, erect image on the same side of the object.

Image distance $v = -10$ cm

Focal length $f = -15$ cm

Object distance $u = ?$

Since $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ (or) $\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$

$$\frac{1}{u} = \frac{1}{-10} - \frac{1}{-15} = \frac{1}{15} - \frac{1}{10} = \frac{-3+2}{30}$$

$$\frac{1}{u} = \frac{1}{-30} \Rightarrow u = -30 \text{ cm}.$$

Thus object distance is 30 cm

Magnification $m = \frac{v}{u} = \frac{-10\text{ cm}}{-30\text{ cm}} = \frac{1}{3} = +0.33$

The positive sign shows shows that the image is erect and virtual. The image is one third of the size of the object.

Q) A 2 cm tall object is placed perpendicular to the principal axis of a convex lens of focal length 10cm. The distance of the object from the lens is 15cm. Find the nature, position and size of the image. Also find the magnification.

Solution:

Height of the object $h = +2$ cm
Focal length $f = +10$ cm
object distance $u = -15$ cm
Image distance $v = ?$
Height of image $h' = ?$

Since $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$

$$\frac{1}{v} = \frac{1}{10} + \frac{1}{-15} = \frac{1}{10} - \frac{1}{15} = \frac{-2+3}{30} = \frac{1}{30}$$

$v = +30$ cm

The positive sign shows that image is formed on other side of the optical mirror. The image is real and inverted.

Magnification $m = \frac{h'}{h} = \frac{v}{u}$

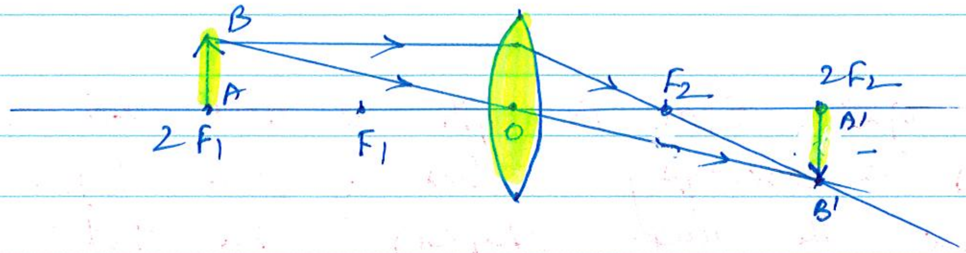
$$h' = \frac{v}{u} \times h = \frac{30}{-15} \times 2 = -4 \text{ cm}$$

$$m = \frac{+30}{-15} = -2$$

The negative sign of m and h' shows that the image is inverted and real.

Q) A convex lens forms a real and inverted image of a needle at a distance of 50 cm from it. Where is the needle placed in front of the convex lens if the image is equal to the size of the object? Also, find the power of the lens.

Solution: If the image is equal to the size of the object, then the object is placed at the centre of curvature



It is given that the image of the needle is formed at a distance of 50 cm from the convex lens.

Object distance $u = -50cm$
Image distance $v = 50cm$
Focal length $f = ?$

Lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{f} = \frac{1}{50} - \frac{1}{-50} = \frac{1}{50} + \frac{1}{50} = \frac{1}{25} \Rightarrow f = 25 cm$
$= 0.25 m$

Power of lens $P = \frac{1}{f} = \frac{1}{0.25} = +4 D$

Q.) Find the power of the concave lens of focal length 2m.

Soln : focal length $f = 2m$

$$\text{Power} = \frac{1}{f} = \frac{1}{-2} = -0.5\,D$$

Here, negative sign arises due to the divergent nature of lens.

Power of given concave lens is $-0.5\,D$.

Q.) An object 5 cm in length is held 25 cm away from a converging lens of focal length 10 cm. Draw the ray diagram and find the position, size and nature of the image.

Soln: Object Distance $u = -25$ cm $v = ?$
Object Height $h = 5$ cm
focal length $f = +10$ cm

Using lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} + \frac{1}{-25} = \frac{1}{10} - \frac{1}{25} = \frac{15}{250}$$

$$v = \frac{250}{15} = +16.67\text{ cm}$$

Positive value shows that image is formed at other side of the lens.

Magnification $m = \frac{v}{u} = \frac{16.67}{-25} = -0.66$

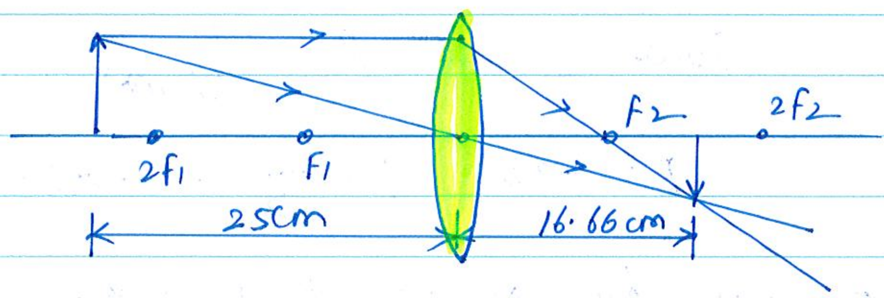
The negative sign shows that the image is real and formed behind the lens.

$$m = \frac{h'}{h} = \frac{h'}{5} = -0.66$$

$$h' = -0.66 \times 5 = -3.3 \text{ cm}$$

The negative value of image height indicates that image is formed is inverted.

The position, size, and nature of image are shown in the following ray diagram.



Q) A concave lens of focal length 15 cm forms an image 10cm from the lens. How far is the object placed from the lens? Draw the ray diagram.

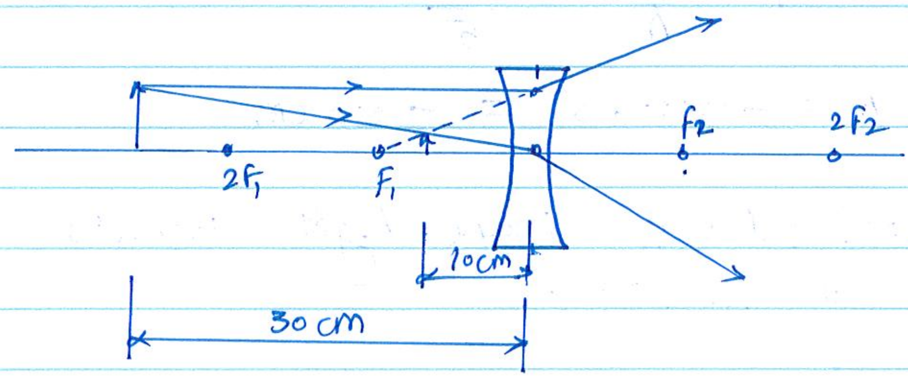
Solution: focal length $f = -15$ cm
Image distance $v = -10$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$\frac{1}{u} = \frac{1}{-10} - \frac{1}{-15} = \frac{-1}{10} + \frac{1}{15} = \frac{-5}{150}$$

$u = -30$ cm

The negative sign indicates that the object is placed 30 cm in front of the lens.



Q) Find the focal length of a lens of power -2.0 D. What type of lens is this?

Solution: Power of lens $P = \frac{1}{f(\text{in m})}$

$P = -2D \quad f = \frac{-1}{2} = -0.5\,m$

A concave lens has negative focal length. Hence, it is a concave lens.

Q) A doctor has prescribed a corrective lens of power $+1.5$ D. Find the focal length of the lens. Is the prescribed lens diverging or converging?

Soln: Power of lens $= \frac{1}{f(\text{in m})}$, $f = \frac{1}{P}$

$f = \frac{1}{1.5} = 0.66\,m$

A convex lens has positive focal length. Hence it is a convex lens or a converging lens.