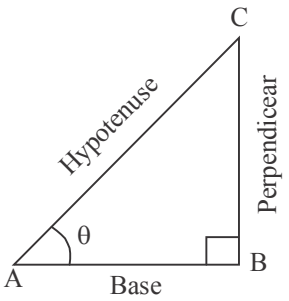
8
Chapter

Trigonometry

Key Points

1. Trigonometric ratio : In ΔABC, $\angle B = 90°$. For $\angle A$,



$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\text{adjacent side}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{Opposite side}}{\text{adjacent side}}$$

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{\text{Hypotenuse}}{\text{adjacent side}}$$

$$\text{cosec } A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\text{Hypotenuse}}{\text{Opposite side}}$$

2. Opposites

$$\sin\theta = \frac{1}{\text{cosec}\,\theta}, \ \text{cosec}\,\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}, \ \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}, \ \cot\theta = \frac{1}{\tan\theta}$$

3. $\tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$

4. Identities

$\sin^2\theta + \cos^2 = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$

$1 + \tan^2\theta = \sec^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1$ and $\sec^2\theta - \tan^2\theta = 1$

$1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$ and $\text{cosec}^2\theta - \cot^2\theta = 1$

5. Trigonometric ratios of some specific angles

$\angle A$	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

6. Trigonometric ratios of complimentary angles

$\sin(90 - \theta) = \cos\theta$

$\cos(90 - \theta) = \text{Sin}\,\theta$

$\tan(90 - \theta) = \cot\theta$

$\cot(90 - \theta) = \tan\theta$

$\sec(90 - \theta) = \text{cosec}\,\theta$

$\text{cosec}(90 - \theta) = \sec\theta$

VERY SHORT ANSWER TYPE QUESTIONS

1. If $\text{Sin}\,\theta = \cos\theta$, find the value of θ

2. If $\tan\theta = \cot(30° + \theta)$, find the value of θ

3. If $\text{Sin}\,\theta = \cos(\theta - 6°)$, find the value of θ

4. If $\cos A = \frac{7}{25}$, find the value of $\tan A + \cot A$

5. If $\tan \theta = \frac{4}{3}$ then find the value of $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}$

6. If $3x = \text{cosec } \theta$ and $\frac{3}{x} = \cot \theta$ then find $3\left(x^2 - \frac{1}{x^2}\right)$

7. If $x = a \sin \theta$ and $y = a \cos \theta$ then find the value of $x^2 + y^2$

8. Find the value of $\text{cosec } 70° - \sec 20°$

9. If $5x = \sec \theta$ and $\frac{5}{x} = \tan \theta$ then find the value of $5\left(x^2 - \frac{1}{x^2}\right)$

10. Find the value of $9 \sec^2 A - 9 \tan^2 A$

11. Express $\sec \theta$ in terms of $\cot \theta$

12. Find the value of $\cos \theta \cos (90 - \theta) - \sin \theta \sin (90 - \theta)$

13. If $\sin (20 + \theta) = \cos 30°$ then find the value of θ.

14. Find the value of $\frac{1 + \tan^2 \theta}{1 + \cot^2 \theta}$

15. Find the value of $\frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$

SHORT ANSWER TYPE (I) QUESTIONS

Prove that :

16. $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$

17. $\sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} = \tan \theta + \text{Sec } \theta$

18. If $x = p \sec \theta + q \tan \theta$ & $y = p \tan \theta + q \sec \theta$ then prove that $x^2 - y^2 = p^2 - q^2$

19. If $7 \sin^2 \theta + 3 \cos^2 \theta = 4$ then show that $\tan \theta = \frac{1}{\sqrt{3}}$

20. If Sin (A – B) = $\frac{1}{2}$, cos (A + B) = $\frac{1}{2}$ then find the value of A and B.

21. Find the value of $\frac{\cos^2 20° + \cos^2 70°}{\sin^2 59° + \sin^2 31°}$.

22. **Prove that :** tan 1° tan 11° tan 21° tan 69° tan 79° tan 89° = 1

23. If sec 4 A = cosec (A – 20°) then find the value of A.

24. If 3 cot A = 4, find the value of $\frac{\text{Cosec}^2 A + 1}{\text{Cosec}^2 A - 1}$.

25. If tan (3x – 15) = 1 then find the value of x.

SHORT ANSWER TYPE QUESTIONS

Prove that :

26. $\frac{\tan A + \text{Sec}\, A - 1}{\tan A - \text{Sec}\, A + 1} = \frac{1 + \text{Sin}\, A}{\text{Cos}\, A}$

27. $\frac{1}{\sec x - \tan x} - \frac{1}{\cos x} = \frac{1}{\cos x} - \frac{1}{\sec x + \tan x}$

28. $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta = \sec \theta \, \text{cosec}\, \theta + 1$

29. $(\sin \theta + \text{cosec}\, \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta$

30. sec A (1 – sin A) (sec A + tan A) = 1

31. If $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$ then show that $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$

32. If tan θ + sin θ = m, tan θ – sin θ = n then show that $m^2 - n^2 = 4\sqrt{mn}$.

33. If $\sec \theta = x + \frac{1}{4x}$, prove that sec θ + tan θ = 2x or $\frac{1}{2x}$

34. If $\sin \theta + \sin^2 \theta = 1$, prove that $\cos^2 \theta + \cos^4 \theta = 1$

35. Without using trigonometric table, the value of
cot θ tan (90 – θ) – sec (90 – θ) cosec θ + $\sin^2 65° + \sin^2 25° + \sqrt{3} \tan 5°$ tan 85°.

36. **Prove that :** $\frac{\cot (90 - \theta)}{\tan \theta} + \frac{\text{cosec}\,(90 - \theta) \sin \theta}{\tan (90 - \theta)} = \sec^2 \theta$

37. Find the value of :

$$\frac{\cos 20^\circ + \cos^2 70^\circ}{\sec^2 50^\circ - \cot^2 40} + 2\,\text{Cosec}^2\,58^\circ - 2\,\text{Cot}\,58^\circ \tan 32^\circ - 4\tan 13^\circ \tan 37^\circ \tan 77^\circ \tan 45^\circ \tan 53^\circ.$$

38. If A, B, C are the angles of ΔABC then prove that $\text{cosec}^2\left(\frac{B+C}{2}\right) - \tan^2\frac{A}{2} = 1$

39. Find the value of $\sec^2 10^\circ - \cot^2 80^\circ + \frac{\sin 15\cos 75 + \cos 15^\circ \sin 75}{\cos\theta \sin(90-\theta) + \sin\theta\cos(90-\theta)}$.

40. Prove that : $\frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} = \tan^2\theta - \cot^2\theta$.

LONG ANSWER TYPE QUESTIONS

Prove That:

41. $\frac{\text{Sec}\,\theta + \tan\theta - 1}{\tan\theta - \text{Sec}\,\theta + 1} = \frac{\text{Cos}\,\theta}{1 - \text{Sin}\,\theta}$

42. $\left(1 + \frac{1}{\tan^2\theta}\right)\left(1 + \frac{1}{\text{Cot}^2\,\theta}\right) = \frac{1}{\text{Sin}^2\,\theta - \text{Sin}^4\,\theta}$

43. $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

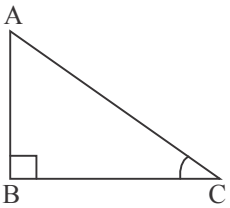
44. $(1 + \cot A + \tan A)(\sin A - \cos A) = \sin A \tan A - \cot A \cos A$

45. If $\text{Sin}\,\theta + \text{Cos}\,\theta = m$ and $\text{Sec}\,\theta + \text{Cosec}\,\theta = n$ then show that $n(m^2 - 1) = 2m$

46. find the value of :

$$\frac{\text{Cot}\,(90-\theta)\,\text{ten}\,\theta - \text{Cosec}\,(90-\theta)\,\text{Sec}\,\theta}{\text{Sin}\,12^\circ\,\text{Cos}\,15^\circ\,\text{Sec}\,78^\circ\,\text{Cosec}75^\circ} + \frac{\text{Cos}^2\,(50+\theta)\tan^2\,(40-\theta)}{\tan 15^\circ \tan 37^\circ \tan 53^\circ \tan 75^\circ}$$

47. In given right triangle if base and perpendicular represents hardwork and success respectively and the ratio between them is 1 : 1 then find $\angle AOB$. Which mathematical concepts has been use in the question? Which values are depicted here?



48. If time bound and continuity are two measurable quantities respectively equal to A & B. If $\text{Sin}(A-B)=\frac{1}{2}$, $\cos(A+B)=\frac{1}{2}$, where $0° < A+B \leq 90°$ find the values of A and B.

49. If $x = \text{Sin}^2\,\theta$, $y = \text{Cos}^2\,\theta$ where *x* & *y* represents honesty and hardwork

(a) What will be the result after joining honesty & hardwork

(b) Which mathematical concept has been used here?

(c) Which values are depicted here?

50. Prove that :

$$\frac{1}{\text{Cosec}\,\theta + \text{Cot}\,\theta} - \frac{1}{\text{Sin}\,\theta} = \frac{1}{\text{Sin}\,\theta} - \frac{1}{\text{Cosec}\,\theta - \text{Cot}\,\theta}$$

51. If $\frac{\text{Cos}\,\alpha}{\text{Cos}\,\beta} = m$ and $\frac{\text{Cos}\,\alpha}{\text{Sin}\,\beta} = n$, then prove that $(m^2 + n^2)\,\text{Cos}^2\,\beta = n^2$

52. If $\tan\theta + \text{Sin}\,\theta = m$. $\tan\theta - \sin\theta = n$, then prove that $m^2 - n^2 = 4\sqrt{mn}$

53. Prove that :

$$\text{Sec}^2\,\theta - \frac{\text{Sin}^2\,\theta - 2\text{Sin}^4\,\theta}{2\text{Cos}^4\,\theta - \text{Cos}^2\,\theta} = 1$$

54. $\text{Cot}\,\theta\,\tan(90° - \theta) - \text{Sec}\,(90° - \theta)\,\text{Cosec}\,\theta + \sqrt{3}\,\tan 12°\,\tan 60°\,\tan 78°$ find its value.

55. Find the value of —

$$\frac{\text{Sec}\,(90° - \theta)\,\text{Cosec}\,\theta - \tan(90° - \theta)\,\text{Cot}\,\theta + \text{Cos}^2\,25° + \text{Cos}^2\,65°}{3\,\tan 27°\,\tan 63°}$$

ANSWERS

1. 45°

2. 30°

3. 24°

4. $\frac{625}{168}$

5. 7

6. $\frac{1}{3}$

7. a

8. 0

9. $\frac{1}{5}$

10. 9

11. $\frac{\sqrt{1+\text{Cos}^2\,\theta}}{\text{Cot}\,\theta}$

12. 0°

13. 50°

14. $\tan^2\theta$

15. $\tan\theta$

20. A = 45°, B = 15°

21. 1

23. 22°

24. $\frac{17}{8}$

25. 20°

35. $\sqrt{3}$

37. – 1

39. 2

46. 0

47. 45° trigonometry, hardwork & success

48. A = 45°, B = 15° honesty, hardwork, Co-operation

49. (a) 1 (b) Trigonometry (c) Honesty & hardwork

54. 2

55. $\frac{2}{3}$