TOPIC-1
Graphical Solution of Linear Equations in Two Variables Consistency/Inconsistency

Quick Review

➢ **Linear Equation in two variables :** An equation of the form $ax + by + c = 0$, where a, b and c are real numbers and a and b are not both zero, is called a linear equation in two variables x and y.

General form of a pair of linear equations in two variables is :

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0,$$

where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers, such that

$$a_1, b_1 \neq 0 \text{ and } a_2, b_2 \neq 0.$$

e.g.,
$$3x - y + 7 = 0,$$
$$7x + y = 3$$

are linear equations in two variables x and y.

➢ There are two methods of solving simultaneous linear equations in two variables :

1. Graphical method,
2. Algebraic method.

1. Graphical Method :

(i) Express one variable say y in terms of the other variable x, $y = ax + b$, for the given equation.

(ii) Take three values of independent variable x and find the corresponding values of dependent variable y, take integral values only.

(iii) Plot these values on the graph paper in order to represent these equations.

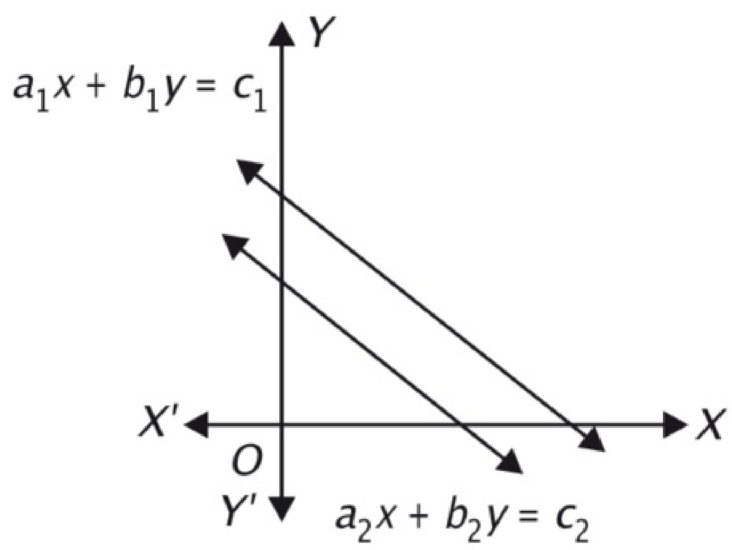
(iv) If the lines intersect at a distinct point, then point of intersection is the unique solution of the two equations. In this case, the pair of linear equations is consistent.

(v) If the lines representing the linear equations coincides, then system of equations has infinitely many solutions. In this case, the pair of linear equations is consistent and dependent.

(vi) If the lines representing the pair of linear equations are parallel, then the system of equations has no solution and is called inconsistent.

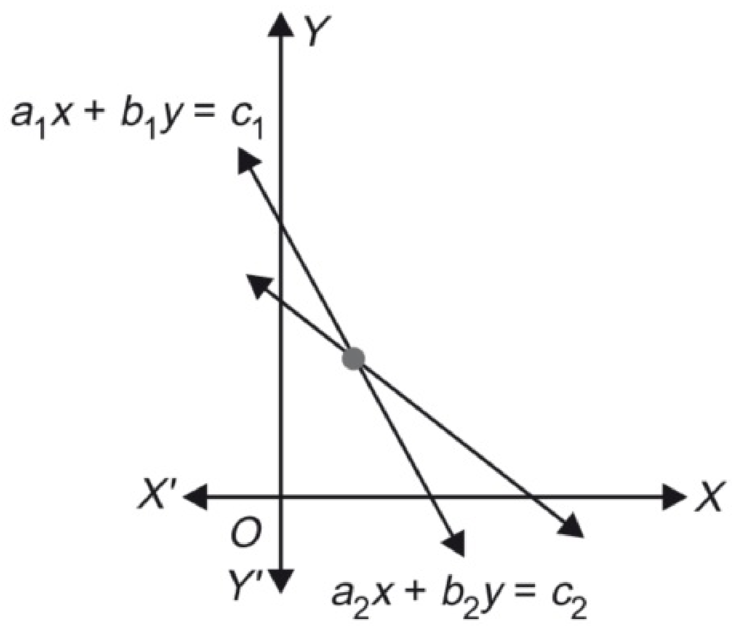
Parallel Lines :

(i) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations is inconsistent with no solution.



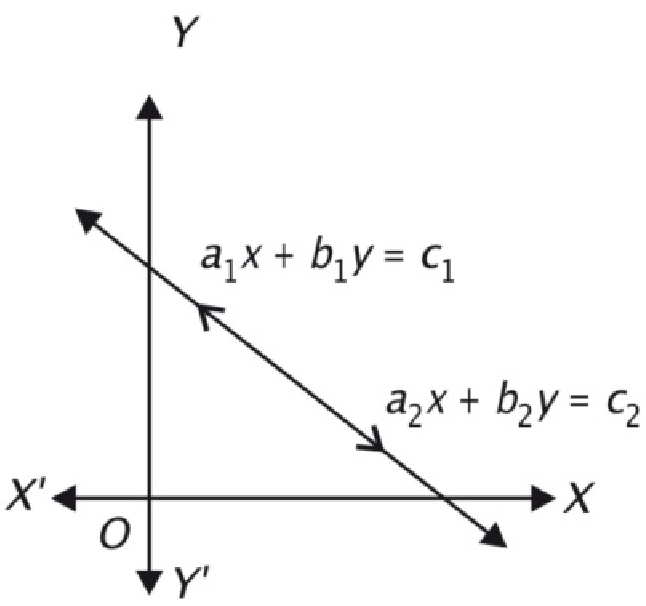
Intersecting Lines :

(ii) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations is consistent with a unique solution.



Coincident Lines :

(iii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of linear equations is consistent with infinitely many solutions.



➢ If $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$,

is a pair of linear equations in two variables x and y such that :

Possibilities of solutions and Inconsistency :

Pair of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratios	Graphical representation	Algebraic interpretation	Conditions for solvability
$x - 2y = 0$ $3x - 4y - 20 = 0$	$\frac{1}{3}$	$\frac{-2}{-4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution Unique solution	System is consistent
$2x + 3y - 9 = 0$ $4x + 6y - 18 = 0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions	System is consistent
$x + 2y - 4 = 0$ $2x + 4y - 12 = 0$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{-4}{-12}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	System is inconsistent

TOPIC-2

Algebraic Methods to solve pair of Linear Equations and Equations reducible to Linear Equations

Quick Review

➤ **Algebraic Method :** We can solve the linear equations algebraically by **substitution method, elimination method** and by **cross-multiplication method**.

1. **Substitution Method :**

(i) Find the value of one variable say y in terms of the other variable *i.e.*, x from either of the equations.

(ii) Substitute this value of y in other equation and reduce it to an equation in one variable.

(iii) Solve the equation so obtained and find the value of x.

(iv) Put this value of x in one of the equations to get the value of variable y.

2. **Elimination Method :**

(i) Multiply given equations with suitable constants, make either the x-coefficient or the y-coefficient of the two equations equal.

(ii) Subtract or add one equation from the other to get an equation in one variable.

(iii) Solve the equation so obtained to get the value of the variable.

(iv) Put this value in any one of the equation to get the value of the second variable.

Note :

(a) If in step (ii), we obtain a true equation involving no variable, then the original pair of equations has infinitely many solutions.

(b) If in step (ii), we obtain a false equation involving no variable, then the original pair of equations has no solution *i.e.*, it is inconsistent.

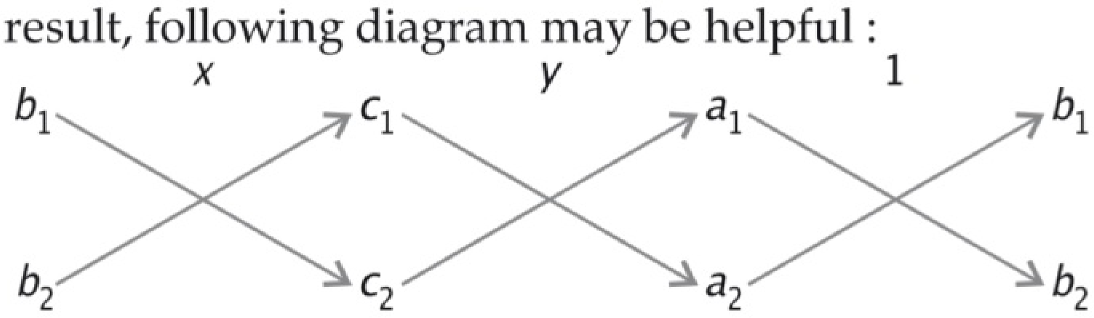
3. **Cross-multiplication Method :** If two simultaneous linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are given, then a unique solution is given by :

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

or

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Note : To obtain the above result, following diagram may be helpful :



The arrows between the two numbers indicate that they are to be multiplied. The product with upward arrows are to be subtracted from the product with downward arrows.

1. If the equations

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

are in the form

$$a_1x + b_1y = -c_1 \text{ and } a_2x + b_2y = -c_2$$

Then, we have by cross-multiplication

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{-1}{a_1b_2 - a_2b_1}$$

➤ **Equations reducible to a Pair of Linear Equations in two variables :** Sometimes, a pair of equations in two variables is not linear but can be reduced to linear form by making some suitable substitutions. Here, first we find the solution of new pair of linear equations and then find the solution for the given pair of equations.

Steps to be followed for solving word problems

S. No.	Problem type	Steps to be followed
1.	Age Problems	If the problem involves finding out the ages of two persons, take the present age of one person as x and of the other as y. Then 'a' years ago, age of 1st person was '$x - a$' years and that of 2nd person was '$y - a$' and after 'b' years, age of 1st person will be '$x + b$' years and that of 2nd person will be '$y + b$' years. Formulate the equations and then solve them.
2.	Problems based on Numbers and Digits	Let the digit in unit's place be x and that in ten's place be y. The two-digit number is given by $10y + x$. On interchanging the positions of the digits, the digit in unit's place becomes y and in ten's place becomes x. The two digit number becomes $10x + y$. Formulate the equations and then solve them.
3.	Problems based on Fractions	Let the numerator of the fraction be x and denominator be y, then the fraction is $\frac{x}{y}$. Formulate the linear equations on the basis of conditions given and solve for x and y to get the value of the fraction.
4.	Problems based on Distance, Speed and Time	We know that Speed $= \frac{\text{Distance}}{\text{Time}}$, $\Rightarrow$ Distance = Speed × Time and Time $= \frac{\text{Distance}}{\text{Speed}}$. To solve the problems related to speed of boat going downstream and upstream, let the speed of boat in still water be x km/h and speed of stream be y km/h. Then, the speed of boat downstream $= x + y$ km/h and speed of boat upstream $= x - y$ km/h.
5.	Problems based on commercial Mathematics	For solving specific questions based on commercial mathematics, the fare of 1 full ticket may be taken as ₹ x and the reservation charges may be taken as ₹ y, so that one full fare $= x + y$ and one half fare $= \frac{x}{2} + y$. To solve the questions of profit and loss, take the cost price of 1st article as ₹ x and that of 2nd article as ₹ y. To solve the questions based on simple interest, take the amount invested as ₹ x at some rate of interest and ₹ y at some other rate of interest.
6.	Problems based on Geometry and Mensuration	Make use of angle sum property of a triangle ($\angle A + \angle B + \angle C = 180°$) in case of a triangle. In case of a parallelogram, opposite angles are equal and in case of a cyclic quadrilateral, opposite angles are supplementary.