Chapter 8 - Motion (IX)

Introduction

Rest : When a body does not change its position with time, we can say that the body is at rest.

Motion : While if a body changes its position with time, it is said to be in motion.

(i) An object is said to be a point object if it changes its position by distances which are much greater than its size.

(ii) Reference Point : A point or some stationary object with respect to which a body continuously changes its position in the state of motion is known as origin or reference point.

Describing Motion

When a tree is observed by an observer A sitting on a bench, the tree is at rest. This is because position of the tree is not changing with respect to the observer A.

Now when same tree T is observed by an observer sitting in a fast moving train or bus with velocity v, then the tree is moving with respect to the observer because the position of tree is changing with respect to the observer B.

Scalers : Physical quantities having only magnitude are called scalar quantities.
Example : Mass, time, distance, speed, work, power, energy, electric charge, area, volume, density, pressure, electric potential, temperature etc.

Vectors : Physical quantities defined with both magnitude and direction are called vector quantities.
Examples : Velocity, acceleration, force, displacement, momentum, weight, torque, electric field, magnetic field etc.

Types of motion

(A) According to Direction
(i) One dimension motion
(ii) Two dimension motion
(iii) Three dimension motion.

(B) According to state of motion

(i) Uniform motion : When a body covers equal distances in equal intervals of time.

Time (sec)	:	0	1	2	3	4	5	6
Distance (mtr)	:	0	10	20	30	40	50	60

Example :
a) An aeroplane flying at a speed of 600 km/h
b) A train running at a speed of 120 km/h.
c) Light energy travelling at a speed of 3×10^8 m/s
d) A spaceship moving at a speed of 100 km/s.

(ii) Non- uniform motion

When a body covers unequal distances in equal intervals of time, the body is said to be moving with a non- uniform motion.

Time (sec) :	0	1	2	3	4	5
Distance (m) :	0	1	4	9	16	21

Examples :

(i) An aeroplane running on a runway before taking off

(ii) A freely falling stone under the action of gravity

(iii) An object thrown vertically upward

(iv) When the brakes are applied to a moving car.

(C) According to path

(i) Linear motion : A body has linear motion if it moves in a straight line or path.

Example : (i) Motion of car moving on a straight road

(ii) motion of ball dropped from the roof of building.

(ii) Circular (or rotational) motion : A body has circular motion if it moves around a fixed point.

A vertical passing through the fixed point around which the body moves is known as axis of rotation.

Example : (a) motion of an electric fan

(b) Motion of merry-go-round

(c) Motion of a spinning top.

(iii) Vibratory motion : A body has vibratory motion if it moves to and fro about a fixed point.
(a) Motion of a pendulum of a wall clock
(b) Motion of a simple pendulum.

Distance and Displacement

(i) Distance : The actual path length between the initial and final positions of the particles gives the distances covered by the particle.

(ii) Displacement : The minimum distance between the initial and final position of a body during that time interval is called displacement.

Analysis :
(i) Distance travelled is a scaler quantity while displacement is a vector quantity.
If a body moves along the circumference of a circle of radius r, then the distance travelled is given by $2\pi r$, while displacement is given by zero.

(ii) When a body continuously moves in the same straight line and in the same direction then displacement will be equal to the distance travelled. But if the body changes its direction while moving then the displacement is smaller than the distance travelled.

$$\text{Displacement} \leq \text{Distance}.$$

Distance	Displacement
(1) It is defined as the actual path transversed by a body	(1) It is the shortest distance between two points which the body moves.
(2) It is a scaler quantity	(2) It is a vector quantity
(3) It can never be negative or zero	(3) It can be negative, zero or positive
(4) Distance can be equal to or greater than displacement	(4) Displacement can be equal to or less than distance.
(5) Distance never decreases with time. For a moving body, it is never zero.	(5) Displacement can decrease with time for a moving body. It can be zero.

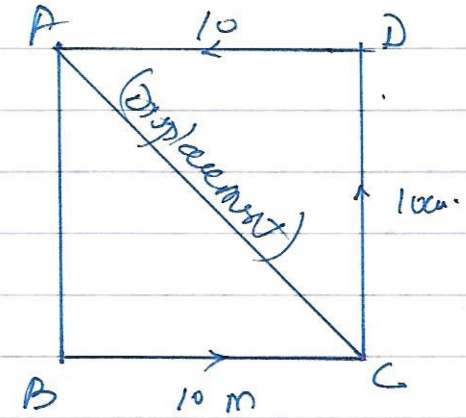
Questions.

(1) An object has moved through a distance. Can it have zero displacement? If yes, support your answer with an example.

ANS) YES, an object which has moved through a distance can have zero displacement.
Example, When a person, walking along a circular path, returns back to the starting point, after completing a circle, his displacement is zero.
But he covers a distance of $2\pi r$, where 'r' is the radius of circular path.
The displacement is zero, as the shortest distance between the initial and final position of the person is zero.

(2) A farmer moves along the boundary of a square field of side 10 m in 40 sec. What will be the magnitude of displacement of the farmer at the end of 2 minutes 20 seconds?

ANS)



The perimeter (boundary) of square field $= 4 \times 10$ m $= 40$ m
Time for moving once $= 40$ sec
Time for journey of farmer $= 140$ sec

No. of times farmer moves $= \frac{140}{40} = 3.5$
For going thrice, displacement $= 0$.

For going half (½), distance $= 10 + 10 = 20$ m.

Displacement AC $= \sqrt{10^2 + 10^2} = 10\sqrt{2}$ m $= 14.14$ m.

3) Which of the following is true for displacement?
(a) It cannot be zero
(b) Its magnitude is greater than the distance travelled by the object.

Ans) a) false, it can be zero

b) false, displacement ≤ distance travelled.

4) Can the displacement of a particle be zero when the distance travelled is not zero?

Ans) Yes, displacement will be zero even if distance is not zero. For example you take a full round circle and comes back to your starting position then your displacement is zero but distance covered is $2\pi r$.

Speed and Velocity

Speed : The distance travelled per unit time is called speed. It is a scaler quantity. Its SI unit is m/s. Speed always remains positive.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \quad (m/s)$$

$$= \frac{S_2 - S_1}{t_2 - t_1} = \frac{\Delta S}{\Delta t}$$ ΔS is distance in time interval Δt.

Velocity : The displacement per unit time is called velocity. It is a vector quantity expressed in m/s. Velocity can be positive, negative or zero.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time Taken}} \quad m/s \quad (ms^{-1})$$

Speed	Velocity
1) Scaler Quantity	1) Vector quantity
2) Rate of distance covered	2) Rate of displacement
3) Cannot be zero for a moving body.	3) Can be zero, +ve or -ve.
4) Speed is velocity without direction	4) Velocity is directed speed.

Types of Speed :

(a) Average and Instantaneous Speed.

Average Speed : $\dfrac{\text{Total distance}}{\text{Total time taken}}$

(i) If a particles travels distance S_1, S_2, S_3 at speeds V_1, V_2, V_3 respectively then,

$$S_{av} = \frac{S_1 + S_2 + S_3}{\frac{S_1}{V_1} + \frac{S_2}{V_2} + \frac{S_3}{V_3}} = \frac{\Sigma S_i}{\Sigma \frac{S_i}{V_i}}$$

(ii) If a particles travels at speeds V_1, V_2 & V_3 for intervals t_1, t_2 etc. then

$$S_{av} = \frac{V_1 t_1 + V_2 t_2 + V_3 V_3 + \cdots}{t_1 + t_2 + t_3 + \cdots} = \frac{\Sigma V_i t_i}{\Sigma t_i}$$

(iii) If a particles moves a distance at speed V_1 and comes back with speed V_2, then average speed $S_{av} = \dfrac{2V_1 V_2}{V_1 + V_2}$

(iv) If a particles moves for two equal time intervals.

$$S_{av} = \frac{V_1 + V_2}{2}.$$

Instantaneous Speed:

The speed of a body at a particular instant of time is called its instantaneous speed.

$$\text{Instantaneous speed} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

(b) Uniform and non-uniform speed:

Uniform Speed: If the speed of the body does not change with respect to time.

Non-Uniform: If the speed of the body is changing with respect to time.

Questions.

1) Distinguish between speed and velocity.

Ans) Refer to notes.

2) Under what condition is the magnitude of average velocity of an object equal to its average speed?

Ans) When the velocity of an object changes at uniform rate i.e. the body is in uniformly accelerated motion. If a body is moving with uniform acceleration.

3) What does speedometer of an automobile measure?

Ans) The speedometer measures the instantaneous speed of the automobile at some particular time.

4) What does the path of an object look like when it is uniform motion?

Ans) The path of an object will be a straight line.

5) During an experiment, a signal from a spaceship reached the ground station in five minutes. What was the distance of the spaceship from the ground station? The signal travels at the speed of light, that is 3×10^8 m/s.

Ans) Speed of light $v = 3\times10^8$ m/s

Time $t = 5$ min $= 5\times60 = 300$.

Distance of spaceship $D =$ speed $\times$ time

$= 3\times10^8$ m/s $\times 300$

$= 9\times10^{10}$ m.

6) Can a body have constant speed but variable velocity?

Ans) Yes. A body in uniform circular motion has constant speed but due to the change in the direction of motion, its velocity changes at every point.

7) What is the simplest type of motion?
Ans) Motion in straight line.

8) What indicates the motion of earth?
Ans) The phenomenon like day and night indicates the motion of the earth.

9) Does the speedometer of a car measure its average speed?
Ans) No. It measure its instantaneous speed.

10) A body is moving with a velocity of 10 m/s. If the motion is uniform, what will be the velocity after 10 sec?
Ans) The velocity remain same 10 m/s since uniform motion.

11) The odometer of a car reads 2000 km at the start of a trip and 2400 km at the end of the trip. If the trip took 8 h. Calculate the average speed of the car in km/h and m/s.
Ans) $S = 2400 - 2000 = 400$ km $t = 8$ h $v = \frac{S}{t} = \frac{400}{8} = 50$ km/h

12) Usha swims in a 90 m long pool. She covers 180 m in one minute by swimming from one end to the other and back along the same path. Find the average speed and average velocity.
Ans) Total distance travelled by Usha in 1 min is 180 m.
Displacement in 1 m = 0 [she came back to original place]
Average Speed $= \frac{S}{t} = \frac{180}{60} = 3$ m/s

Average velocity $= \frac{S}{t} = \frac{0}{t} = 0$ m/s

Acceleration (Rate of change of velocity)

The rate at which the velocity change is called acceleration. It is a vector quantity. Its SI unit is m/s^2 or ms^{-2}

$$\therefore \quad a = \frac{v-u}{t}$$

Deceleration or Retardation : If the velocity is (-ve) i.e. if velocity of a body decreases, the acceleration is called retardation or deceleration.

Uniform accelerated motion : When the change in velocity is same in equal time intervals, the motion called uniformly accelerated motion, otherwise, it is non-uniformly accelerated motion.

average acceleration $a_{av} = \dfrac{a_1t_1 + a_2t_2}{(t_1 + t_2)}$.

Questions

1) When will you say a body is in
(i) Uniform acceleration
(ii) Non-uniform acceleration.

Ans) (i) A body is in uniform acceleration when equal changes in velocity take place in equal intervals of time, however small these intervals may be.

(ii) A body is said to be possessing non-uniform acceleration when unequal changes in velocity takes in equal intervals of time, however small these intervals may be.

2) A bus decreases its speed from 80 km/h to 60 km/h in 5 sec. Find the acceleration of the bus.

Ans) Given $t = 5$ sec

Initial speed $u = 80 \text{ km/h} = 80 \times \frac{5}{18} = \frac{400}{18}$ m/s

Final speed of the bus $v = 60 \text{ km/h} = 60 \times \frac{5}{18} = \frac{300}{18}$ m/s

Acceleration $a = \frac{v-u}{t} = \left(\frac{300}{18} - \frac{400}{18}\right) \div 5$

$= \frac{-100}{18} \div 5 = \frac{-100}{18} \times \frac{1}{5} = \frac{-20}{18}$

$a = -1.11 \text{ m/s}^2$ Ans.

3) A train starting from a railway station and moving with uniform acceleration attain a speed 40 km/h in 10 minute. Find its acceleration.

Ans) Given $t = 10 \times 60 = 600$ sec

$u = 0$ (since train start from rest)

$v = 40 \times \frac{5}{18} = \frac{200}{18}$ m/s

$$54 \overline{)100} = 1.01, \quad 100 - 54 = 6$$

$a = \frac{v-u}{t} = \frac{200}{18} \div 600 = \frac{200}{18} \times \frac{1}{600}$

$a = \frac{1}{18 \times 3} = \frac{1}{54} = 0.0185$ m/s.

$\therefore a = 0.0185$ m/s Ans.

4) When is the acceleration taken as negative?

Ans) Acceleration is taken as negative if it is in direction opposite to the direction of velocity.

5) Starting from a stationary position, Rahul paddles his bicycle to attain a velocity of 6 m/s in 30 sec. Then he applies brakes such that the velocity of the bicycle comes down to 4 m/s in next 5 sec. Calculate the acceleration of the bicycle in both case.

Ans) Case - 1 : $u = 0$, $v = 6$ m/s

$t = 30$ sec

$a = \frac{v-u}{t} = \frac{6-0}{30} = \frac{1}{5}$

$a = 0.2$ m/s

Case - 2 : $u = 6$ m/s $v = 4$ m/s

$t = 5$ sec

$a = \frac{v-u}{t} = \frac{4-6}{5} = \frac{-2}{5}$

$a = -0.4$ m/s^2

Equation of Motion

Motion under uniform acceleration

Suppose a body starts with initial velocity (u), moving with an acceleration attains a velocity (v) after time (t) travels a distance (s),

u - initial velocity
v - final velocity
t - time taken
s - distance travelled

then motion can be described by following equation.

a) $v = u + at$

b) $s = ut + \frac{1}{2}at^2$

c) $v^2 - u^2 = 2as$.

Derivation of equation of motion

(i) $\boxed{v = u + at}$

Let u - initial velocity v - final velocity
t - time taken

then acceleration = $\dfrac{\text{change in velocity}}{\text{Time Taken}}$

$$a = \frac{v - u}{t}$$

$$\therefore \quad v = u + at$$

(ii) $$\boxed{s = ut + \frac{1}{2}at^2}$$

Let
s - displacement
t - time taken
a - uniform acceleration
u - initial velocity
v - final velocity

$\therefore$ Average velocity $V_{av} = \frac{u+v}{2}$

Displacement $s = V_{av} \times t$

$$s = \left(\frac{u+v}{2}\right) t$$

Substituting $v = u + at$, we get

$$s = \left(\frac{u + u + at}{2}\right) t$$

$$s = \frac{(2u + at)}{2} t$$

$$s = ut + \frac{1}{2}at^2$$

(iii) $$\boxed{v^2 - u^2 = 2as}$$

Velocity at any time, $v = u + at$

$s = \left(\frac{u+v}{2}\right) t$ $\quad a = \frac{v-u}{t}$ $\quad t = \frac{v-u}{a}$

$$s = \frac{(u+v)}{2} \frac{(v-u)}{a}$$

$$\therefore \quad v^2 - u^2 = 2as$$

(A) These equations are applicable only for constant acceleration or uniform acceleration.

(B) The equations of motion under gravity.

Acceleration due to gravity (g) : The acceleration which is gained by an object because of the gravitational force is called its acceleration due to gravity.
It has standard value, $g = 9.80665 \; m/s^2$

When the body is coming towards the centre of earth

(a) $v = u + gt$

[Replace a by g]

(b) $h = ut + \frac{1}{2}gt^2$

(c) $v^2 - u^2 = 2gh$.

When body is thrown upwards against gravity, then a retardation is produced due to attraction of the earth. In equation of motion, "a" is replaced by (−g) and thus equation become.

a) $v = u - gt$

b) $h = ut - \frac{1}{2}gt^2$

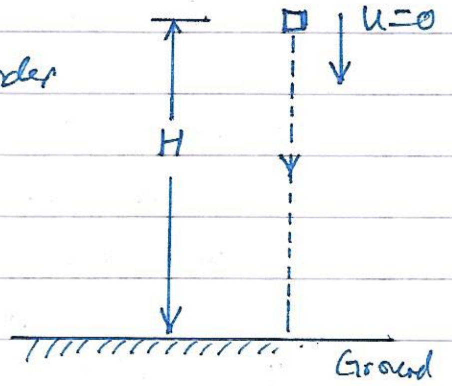
c) $v^2 - u^2 = -2gh$

c) Distance covered by a body in n^{th} sec i.e.

$$S_n = u + \frac{1}{2}a(2n-1)$$

Body falling freely under Gravity

A body released near the surface of the earth is accelerated downward under the influence of force of gravity.



(a) Time of flight

$$t = \sqrt{\frac{2H}{g}}$$

$S = ut - \frac{1}{2}gt^2$

$-H = -\frac{1}{2}gt^2$

$t = \sqrt{\frac{2H}{g}}$

(when height is given)

(b) Final velocity when body reaches the ground.

$v^2 - u^2 = 2gH$

$u = 0 \quad v = V$

$V^2 = 2gH$

$V = \sqrt{2gH}$

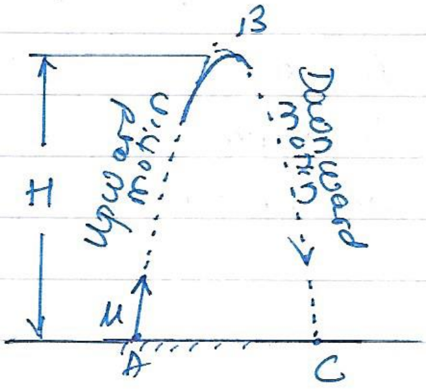
(i) $h = \frac{1}{2}gt^2$ i.e. $h \propto t^2$ [$\because$ g is constant]

Distance falling in time t, 2t, 3t etc. will be in ratio of $1^2 : 2^2 : 3^2 : \ldots$ i.e. squares of integers.

(ii) The distance fallen in n^{th} sec $= \frac{1}{2}g(2n-1)$

So distance fallen in 1st, 2nd, 3rd sec will be in ratio 1 : 3 : 5 i.e. odd integers only.

Body is projected vertically up



It includes two types of motion

(i) Decelerated motion from A to B because the direction of velocity and acceleration is opposite. So speed decreases.

(ii) Accelerated motion from B to C because the direction of velocity and acceleration is same (downward). So speed increases.

(a) Time of flight

Time taken to reach maximum height H.

$v = u - gt$

v is 0 at max height

$\therefore \; u - gt = 0, \quad t = \frac{u}{g}$

time of flight $T = 2t$ (to reach ground again), $\boxed{T = \frac{2u}{g}}$

(b) Maximum Height

Using $v^2 - u^2 = -2gH_{max}$

$-u^2 = -2gH_{max}$ [$\because v = 0$ at max height]

$H_{max} = \frac{u^2}{2g}$

(c) Final Velocity

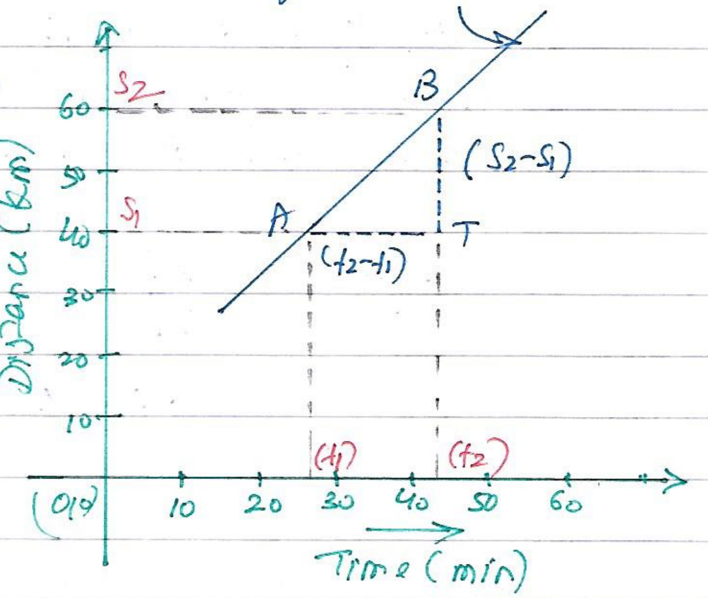
$v = u - gt \qquad v_f = u - g\left(\frac{2u}{g}\right) \qquad \left[t = \frac{2u}{g}\right]$

$v_f = -u$ i.e. the body reaches the ground with same speed with which it was thrown vertically upwards.

Displacement - Time Graph

The change in the position of an object with time can be represented on the distance - time graph.
Time is taken along x-axis.
Distance is taken along y-axis.

a) For uniform speed, a graph of distance travelled against time is a straight line.



We can use distance-time graph to determine the speed of the object.

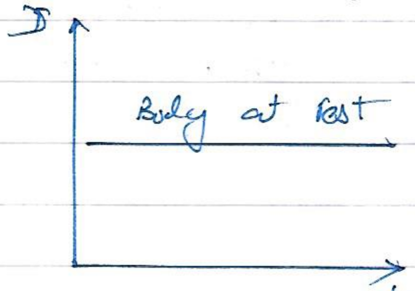
The slope of distance - time graph gives the velocity of motion

$$\text{Slope} = \frac{BT}{AT} = \frac{\text{Displacement}}{\text{Time}} = \frac{S_2 - S_1}{t_2 - t_1}$$

$$\therefore V = \frac{S_2 - S_1}{t_2 - t_1} = \text{slope of the graph.}$$

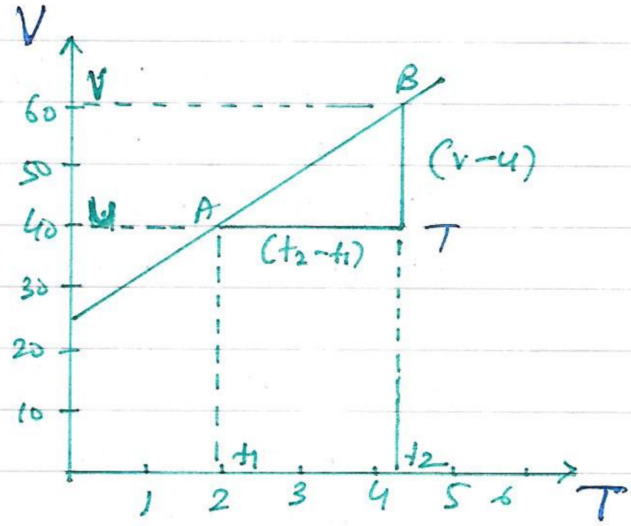
b) For non-uniform motion, slope is a curved line

c) For body at rest (stationary body), slope is a straight line parallel to time axis.



Velocity - Time Graph

Time is taken on x-axis and velocity is taken on y-axis.

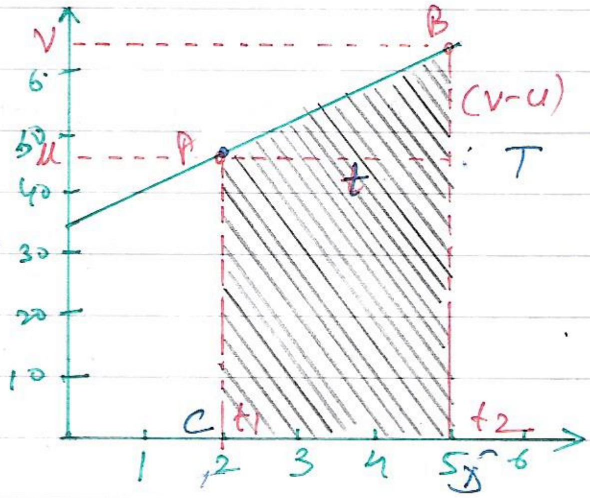


1) The slope of velocity time (v-t) graph gives the acceleration of motion

$$\text{Slope} = \frac{BT}{AT} = \frac{(v-u)}{t_2 - t_1} = \frac{v-u}{t} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

$$\therefore a = \frac{v-u}{t} = \text{Slope of v-t graph.}$$

(2) Area under (v-T) curve



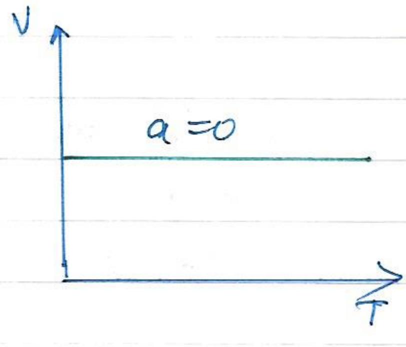
Area under (v-T) graph gives us distance/displacement.

Area of ABCD $= \frac{1}{2} t (u+v)$
[Area of Trapezium]

$$\text{Area ABC} = \frac{u+v}{2} \times t = V_{av} \times t = \text{Distance/Displacement}$$

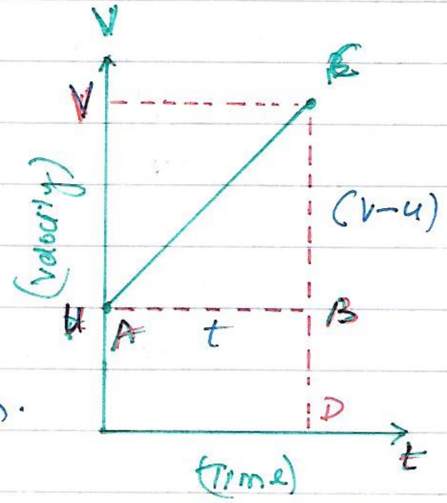
$$\therefore \text{Displacement} = \frac{v+u}{2} \times t = \text{area under curve.}$$

(3) For line parallel to time axis. Velocity is constant. Therefore, acceleration is zero.



Equations of Motion – Graphical Method

(1) Velocity - Time Equation ($v = u + at$)



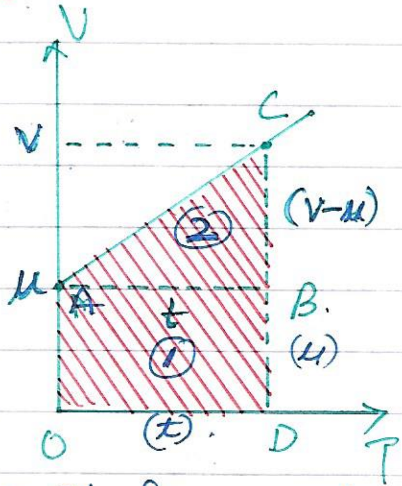
Consider the v-t graph shown for a body having velocity u at $t=0$ and v at t seconds.

Slope of (v-t) graph gives acceleration.

$$\therefore \; a = \text{Slope} = \frac{BC}{AB} = \frac{v-u}{t}$$

$$v - u = at \quad \text{or} \quad v = u + at$$

(2) Position - Time equation ($S = ut + \frac{1}{2}at^2$)



Consider a v-t graph for a body having velocity u at $t=0$ and v at t sec.

Displacement = Area (OACBD)

= Area(rectangle) + Area(Triangle)

$$S = t \times u + \frac{1}{2} \times t \times (v-u)$$

$$S = ut + \frac{1}{2}(v-u) \times t \qquad \text{But } (v-u) = at \; [\because v = u + at]$$

$$S = ut + \frac{1}{2}at^2$$

(3) Position - velocity equation: ($v^2 - u^2 = 2as$)

Displacement, S = Area of Trapezium $= \frac{1}{2} \times t \times (v+u)$

$$t = \frac{v-u}{a} \quad \therefore \; S = \frac{1}{2}\frac{(v+u)(v-u)}{a}$$

$$\therefore \; (v+u)(v-u) = 2as, \qquad v^2 - u^2 = 2as$$

Questions - 1

1) What are the uses of a distance time graph?

Ans) a) It tells us about the position of the body at any instant of time.

b) From the graph, we can see the distance covered by the body during a particular interval of time.

c) It also gives us information about the velocity of the body at any instant of time.

$$V = \frac{S_2 - S_1}{t_2 - S_1}$$

2) A train starting from rest attains a velocity of 72 km/h in 5 minutes. Assuming acceleration is uniform, find: (i) the acceleration and (ii) distance travelled.

Ans) $u = 0$ (Start from rest) $v = 72 \times \frac{5}{18} = 20$ m/s $t = 5 \times 60 = 300$ s

(i) $v = u + at$, $a = \frac{v}{t} = \frac{20}{300} = \frac{1}{15} = 0.067$ m/s^2

(ii) Using $v^2 - u^2 = 2as$, $v^2 = 2aS$, $S = \frac{v^2}{2a} = \frac{20 \times 20 \times 15}{2 \times 1}$

$S = 3000$ m $= 3$ km.

3) A car accelerates uniformly from 18 km/h to 36 km/h in 5 sec. Calculate (i) acceleration (ii) the distance covered.

Soln) $u = 18 \times \frac{5}{18} = 5$ m/s, $v = 36 \times \frac{5}{18} = 10$ m/s $t = 5$ s

(i) $a = \frac{v-u}{t} = \frac{10-5}{5} = \frac{5}{5} = 1$ m/s^2.

(ii) $S = ut + \frac{1}{2}at^2 = 5 \times 5 + \frac{1}{2} \times 1 \times 5 \times 5 = 25 + 12.5 = 37.5$ m

4) The brakes are applied to a car produces an acceleration of 6 m/s^2 in the opposite direction. If car takes 2 sec to stop after brakes are applied, calculate distance travelled. [12 m]

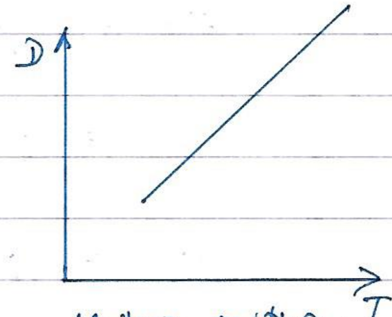
[Hint: $v = u + at$, $v = 0$, find u. $S = ut + \frac{1}{2}at^2$]

$a = -6$ m/s^2, $t = 2$ sec.

Questions – 2

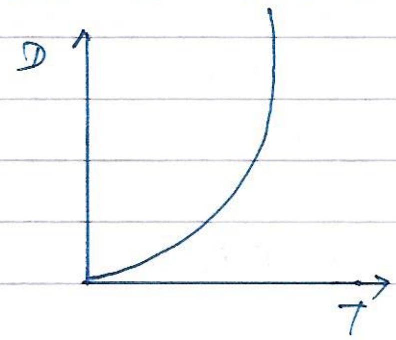
1) What is the nature of the distance time graph for uniform and non-uniform motion of an object?

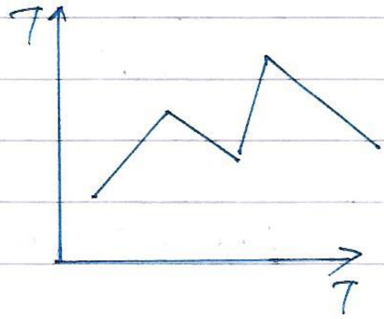
Ans) Uniform motion:
The distance time-graph for uniform motion is a straight line not parallel to the time axis



Uniform motion.

Non-uniform motion

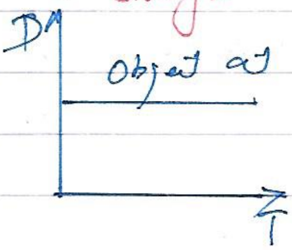




The distance time-graph for non-uniform motion is not a straight line. It can be a curve or a zigzag line not parallel to the time axis.

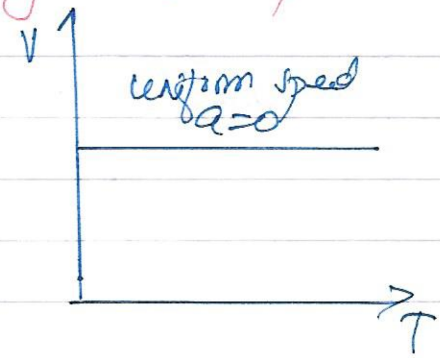
2) What can you say about the motion of an object whose distance time graph is a straight line parallel to the time axis

Ans) The object is stationary.

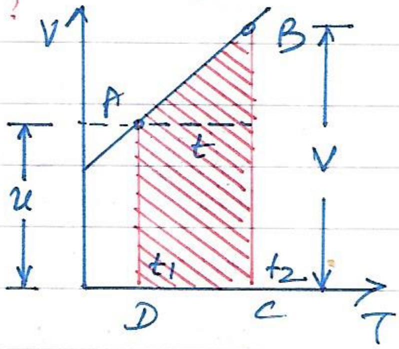


3) What can you say about the motion of an object if its speed-time graph is a straight line parallel to the time axis?

Ans) The object has uniform speed
Acceleration is zero.



4) What is the quantity which is measured by the area occupied below the velocity - time graph?



Ans) Displacement.

Area of trapezium $= \frac{1}{2} h (a+b)$

$= \frac{1}{2} \times t \times (v+u)$

$A = \frac{1}{2}(v+u)t$ = Displacement.

5) A bus starting from rest moves with a uniform acceleration of $0.1\ m/s^2$ for 2 minute. Find (a) the speed acquired (b) the distance travelled.

Ans) $u = 0$ (Since bus starts from rest)

$v = ?$

$a = 0.1\ m/s^2$ $\quad t = 2\ min = 2 \times 60 = 120\ sec$

$s = ?$

(i) $v = u + at$, $v = 0 + 0.1 \times 120$, $v = 12\ m/s$

(ii) $s = ut + \frac{1}{2}at^2$, $s = 0 + \frac{1}{2}\ 0.1 \times 120 \times 120 = 720\ m$

Therefore, Final speed acquired $v = 12\ m/s^2$

Distance travelled $s = 720\ m$.

6) A train is travelling at a speed of 90 km/h. Brakes are applied so as to produce a uniform acceleration of $-0.5\ m/s^2$. Find how far the train will go before it is brought to rest.

Ans) $u = 90 \times \frac{5}{18} = 25\ m/s$ $\quad v = 0$, $a = -0.5\ m/s^2$

$v^2 - u^2 = 2as$, $\quad 0 + 25 \times 25 = +2 \times \frac{5}{10} \times s$

$s = 625\ m$

7) A trolley, while going down an inclined plane, has an acceleration of 2 cm/s². What will be its velocity 3 sec after the start?

Ans) $u = 0$, $v = ?$ $t = 3$ sec, $a = 2\ cm/s^2$

$v = u + at$, $v = 0 + 2 \times 3$, $v = 6\ cm/s$

8) A racing car has uniform acceleration of 4 m/s². What distance will it cover in 10 sec after start.

Ans)

$u = 0$, $a = 4\ m/s^2$ $t = 10$ s.

$S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 4 \times 10 \times 10$

$S = 200\ m$

9). A stone is thrown in a vertically upward direction with a velocity of 5 m/s. If the acceleration of the stone during its motion is 10 m/s² in the downward direction, what will be the height attained by the stone and how much time will it take to reach there?

Ans) $u = 5\ m/s$ $a = -10\ m/s$ (stone is thrown upward)

$v = 0$ (at max height speed is zero)

$S = ?$ $t = ?$

$v = u + at$

$0 = u - at$

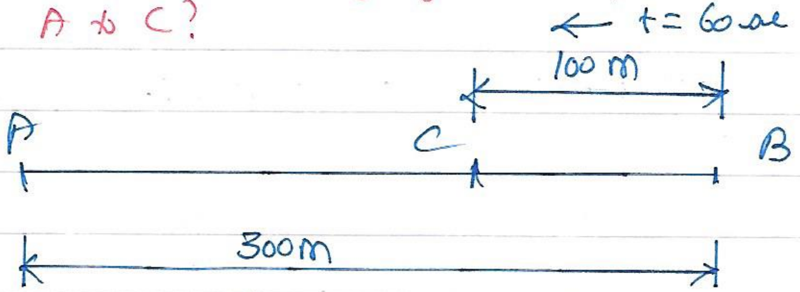
$t = \frac{u}{a} = \frac{5}{10} = 0.5$ sec

$v^2 - u^2 = 2aH$, $H = \frac{-u^2}{2a}$

$H = -\frac{5 \times 5}{2(-10)} = \frac{5}{4} = 1.25\ m$

10) Joseph jogs from one end A to the other end B of a straight 300 m road in 2 minutes 50 seconds and then turns around and jogs 100 m back to point C in another 1 minute. What are Joseph's average speed and velocities in jogging (a) from A to B and (b) from A to C?

Ans.)



$t = 2\text{ min} + 50\text{ sec}$

$= 170\text{ sec}$

(a) <u>from A to B</u>

Distance covered = 300 m

Time taken = 170 s

Average speed $V_{av} = \frac{s}{t} = \frac{300}{170} = 1.76$ m/s.

Average velocity $V_{av} = \frac{\text{Displacement}}{\text{Time}} = \frac{300}{170} = 1.76$ m/s

Average speed and velocities are same.

(b) <u>A to C</u>

Distance covered $S = 300 + 100 = 400$ m

Time taken $t = 170 + 60 = 230$ sec

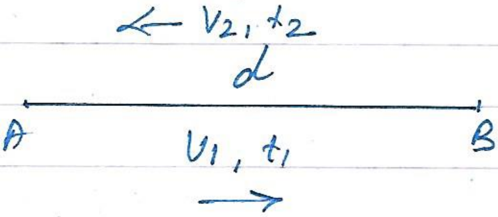
Average speed $V_{av} = \frac{S}{t} = \frac{400}{230} = 1.74$ m/s.

Displacement A to C = 200 m

Average velocity $V_{av} = \frac{200}{230} = 0.869$ m/s

(11) Abdul while driving to school computes the average speed for his trip to be 20 km/h. On his return trip along the same route, there is less traffic and the average speed is 40 km/h. What is the average speed for Abdul's trip?

Ans) Let one way distance be d.
Let t_1 be time for trip from home to school, and t_2 be the time taken for his return trip



$v_1 = 20$ km/h
$v_2 = 40$ km/h

$$t_1 = \frac{d}{v_1} \qquad t_2 = \frac{d}{v_2}$$

$$\text{Average speed} = \frac{d_1 + d_2}{t_1 + t_2} = \frac{d + d}{\frac{d}{v_1} + \frac{d}{v_2}} = (2d) \div \left(\frac{d}{v_1} + \frac{d}{v_2}\right)$$

$$\text{Average Speed} = 2d \div \frac{dv_2 + dv_1}{v_1 v_2} = 2d \div \frac{d(v_1 + v_1)}{v_1 v_2}$$

$$\text{Average Speed} = 2d \times \frac{v_1 v_2}{d(v_1 + v_2)} = \frac{2 v_1 v_2}{v_1 + v_2}$$

$$V_{av} = \frac{2 \times 20 \times 40}{20 + 40} = \frac{80}{3}$$

$$V_{av} = 26.6 \text{ km/h}$$

[Average Speed = $\frac{\text{Total Distance Travelled}}{\text{Total time taken}}$]

$$S_{av} = \frac{s_1 + s_2 + s_3 + \cdots}{t_1 + t_2 + t_3 + \cdots}$$

12) A motorboat starting from rest on a lake accelerates in a straight line at a constant rate of $3\ m/s^2$ for 8 sec. How far does the boat travel during this time?

Ans) $u = 0$ (Body start from rest)

$a = 3\ m/s^2$

$t = 8$ sec $S = ?$

$S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} 3 \times 8 \times 8$

$S = 3 \times 4 \times 8 = 12 \times 8$

$S = 96$ m Ans.

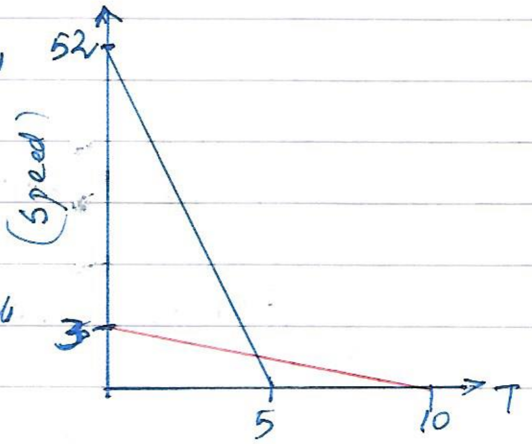
13) The driver of a car travelling at 52 km/h applies the brakes and accelerates uniformly in the opposite direction. The car stops in 5 s. Another driver at 3 km/h in another car applies his brakes slowly and stops in 10 sec. On the same graph paper, plot the speed versus time graph for the two cars. Which cars travelled farther after the brakes were applied?

Ans

(i) $V = 52\ km/h = 52 \times \frac{5}{18} = 14.4\ m/s$

$S = \frac{1}{2} \times 5 \times 14.4$

$S = 36$ m.



(ii) $V = 3\ km/h = 3 \times \frac{5}{18} = 0.83\ m/s$

$S = \frac{1}{2} \times 10 \times 0.83 = 4.15$ m

∴ The car moving at 52 km/h travels more distance.

14) A ball is gently dropped from a height of 20 m. If its velocity increases uniformly at the rate of 10 m/s with what velocity will it strike the ground? After what time will it strike the ground?

ANS) $u = 0$

$v = ?$

Distance $H = 20$ m Acceleration $a = 10\ m/s^2$

$t = ?$

$$v^2 - u^2 = 2aH$$

$$v^2 = 2aH$$

$$v = \sqrt{2gH} = \sqrt{2 \times 10 \times 20} = \sqrt{20 \times 20}$$

$$v = 20\ m/s$$

$$v = u + at \quad \Rightarrow v = at \qquad t = \frac{v}{a} = \frac{20}{10} = 2\ sec$$

$$\therefore t = 2\ sec$$

UNIFORM CIRCULAR MOTION

An athlete runs along the circumference of a circular path. This type of motion is known as circular motion. The movement of an object in a circular path is called circular motion.

When object moves in a circular path with a constant velocity, it is motion is called uniform circular motion. In uniform circular motion, the magnitude of the velocity is constant at all points and direction of the velocity changes continuously.

We know that the circumference of a circle of radius r is given by $2\pi r$. If the athlete takes t seconds to go once around the circular path of radius r, the velocity v is given by

$$v = \frac{2\pi r}{t}$$

Questions

1) An artificial satellite is moving in a circular orbit of radius 42250 km. Calculate its speed if it takes 24 hours to revolve around the earth.

Sol) $R = 42250$ km

$t = 24$ hrs $\qquad v = ?$

$$v = \frac{2\pi r}{t} = \frac{2 \times 3.14 \times 42250}{24}$$

$= 11055.4$ km/h

$= 3070.9$ m/s

$V = 3.07$ km/s