Chapter 9
Forces and laws of motion

introduction

What is force?

An influence which changes or tends to change the state of rest or of uniform motion, direction of motion or shape or size of body.
It is the force that enables us to do any work.
To do anything, either we pull or push the object.
Therefore, pull or push is called force.

Example, to open a door, either we push or pull it.
A drawer is pulled to open and pushed to close.

Effect of force :

(i) Force can make a stationary body in object. For example, a football can set to move by kicking it.

(ii) Force can stop a moving body. For example, by applying brakes, a running cycle or a running vehicle can be stopped.

(iii) Force can change the direction of moving object. For example, by moving steering the direction of a running vehicle is changed.

(iv) Force can change the speed of a moving body. By accelerating, the speed of a running vehicle can be increased. or by applying breaks the speed of a running vehicle can be decreased.

(v) Force can change the shape and size of an object. For example, by hammering, a block of metal can be turned into a thin sheet. By hammering a stone can be broken into pieces.

Balance and Unbalanced Forces.

Balanced Forces :

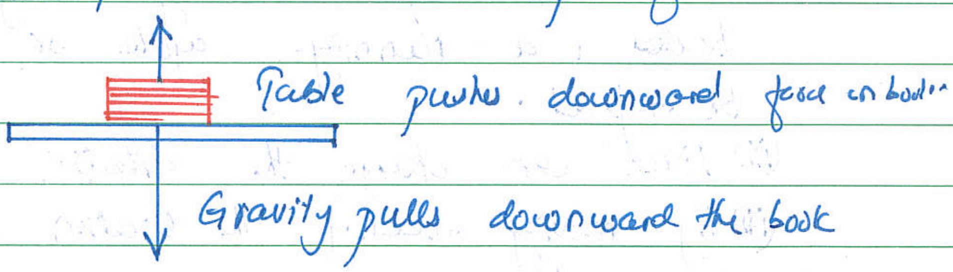
If the resultant of applied forces is equal to zero, it is called balanced forces.

Example, in the tug of war if both the team apply similar magnitude of forces in opposite directions, rope does not move in either side. This happen because of balanced forces in which resultant of applied forces become zero.

Balanced forces do not cause any change of state of an object. Balanced forces are equal in magnitude and opposite in direction.

Balanced forces can change the shape and size of an object. For example, when forces are applied from both sides over a balloon, the size and shape of balloon is changed.

The force of book are balanced.



UnBalanced Forces :

If the resultant of applied forces are greater than zero, the forces are called unbalanced forces. An object in rest can be moved because of applying balanced forces. Unbalanced forces can do the following :

(i) Move a stationary object (ii) Increase the speed of moving object (iii) Decrease the speed of moving object (iv) Stop moving object (v) Change the shape and size of the object.

Laws of Motion

Galilo Galilei : Galileo first of all said that object move with a constant speed when no forces act on them. This means that if an object is moving on a frictionless path and no other forces is acting upon it, the object would be moving forever. That is, there is no unbalanced force working on the object.

- But practically it is not possible for any object. Because to attain the condition of zero, unbalanced forces is impossible. Force of friction, force of air and many other forces are always acting upon an object.

Newton's laws of motion

Newton studied the ideas of Galileo and gave the three laws of motion. These laws are known as Newton's laws of motion.

Newton's First Law of Motion (Law of Inertia)

"An object at rest will stay at rest, and an object in motion will stay in motion at constant velocity, unless acted upon by an unbalanced force. Example, Book lying on table will only move when a force is applied on it by hand."

Newton's first Law of Motion in Everyday Life :

(a) A person standing in a bus falls backward when bus starts moving suddenly. This happens because the person and bus both are in rest while bus is not moving, but as the bus starts moving, the legs of the person start moving along with the bus but rest portion of the body has the tendency to remain in rest. Because of this, the person falls backward; if he is not alert.

(b) A person standing in a moving bus falls forward if driver applies brakes suddenly. This happens because when bus is moving, the person standing in it is also in motion along with bus. But when driver applies brakes the speed of bus decreases suddenly or bus comes in the state of rest suddenly, in this condition the legs of the person which are in contact with the bus comes in rest while the rest part of the body have the tendency to remain in motion. Because of this person falls forward if he is not alert.

Mass and Inertia

Inertia : Property of the body by virtue of which it opposes any change in its state of rest or uniform motion in a straight line.

Mass : Mass is the measure of interia.

This means, larger mass, greater is the intertia. Since a heavy object has more inertia, thus it is difficult to push or pull a heavy box over the ground than lighter one. (4)

Inertia Rest :

Due to this inertia, a body at rest tends to remain at rest. Eg. when a bus suddenly starts moving forward, passengers in the bus fall backward.

Inertia of motion:

Due to this inertia, a body in uniform motion tends to continue its motion. Eg. Passengers fall forward when a fast moving bus stops suddenly.

Momentum

Momentum is the power of motion of an object.
The product of velocity (v) and mass (m) is called momentum.
It is denoted by 'p'
Thus, Momentum of the object = Mass x Velocity

$$p = m \times v$$

Consider the following explanations:

(a) During the game of table tennis if the ball hits a player it does not hurt him. On the other hand, when a fast moving cricket ball hits a spectator, it may hurt. Cricket ball has more momentum than tennis ball.

(b) Even a small bullet is able to kill a person when it is fired from the gun because of its momentum due to great velocity.

(c) A person get injured severely when hit by a moving vehicle because of momentum of vehicle due to mass and velocity.

Momentum has both direction and magnitude. Its direction is the same as that of velocity v. SI unit is $kg\,ms^{-1}$

Since, momentum = mass × velocity

$$p = m \times v$$

when velocity is zero, body is at rest.

then $p = 0$

Thus, the momentum of an object in the rest i.e. non-moving, is equal to zero.

Numerical Problems based on Momentum

Example-1: What will be the momentum of a stone having mass of 10 kg when it is thrown with a velocity of 2 m/s?

Solution:

Mass (m) = 10 kg

Velocity (v) = 2 m/s

Momentum (p) = mass(m) × velocity(v)

$$p = 10 \times 2 \; kg\,ms^{-1}$$

$$p = 20 \; kg\,ms^{-1}$$

Example-2: Calculate the momentum of a bullet of 25 g when it is fired from a gun with velocity of 100 m/s.

Solution: $m = 25\,g$ $v = 100\,m/s$

$= 0.025\,kg$ $p = mv = 0.025 \times 100$

$$p = 2.5\,kg\,m/s$$

∴ Thus, momentum of bullet = 2.5 kg m/s.

Example 3: Calculate the momentum of a bullet having mass 25 g is thrown using hand with a velocity of 0·1 m/s.

Solution: $v = 0·1$ m/s $\quad m = 25$ g $= 0·025$ kg

Momentum $p = m \times v$

$= 0·025 \times 0·1$

$= 0·0025$ kg m/s

Thus, the momentum of the bullet $= 0·0025$ kg m/s.

Example 4: The mass of a goods lorry is 4000 kg and the mass of goods loaded on it is 20,000 kg. If the lorry is moving with a velocity of 2 m/s, what will be its momentum?

Solution: $v = 2$ m/s

Total mass (m) $= 4000$ kg $+ 20,000$ kg

$= 24,000$ kg

$p = m \times v = 24,000 \times 2$

$= 48,000$ kg ms^{-1}

the momentum of the lorry $= 48,000$ kg ms^{-1}

Example 5: A car having mass of 1000 kg is moving with a velocity of 0·5 m/s. What will be its momentum?

Solution: $v = 0·5$ m/s $\quad m = 1000$ kg

$p = m \times v = 1000 \times 0·5$

$p = 500$ kg m/s

Second law of motion

Since the application of an unbalanced force brings a change in the velocity of the object, it is therefore clear that a force also produces a change in momentum.

Force necessary to change the momentum of an objects depends on the time rate at which the momentum is changed.

The second law of motion states that "the rate of change of momentum of an object is proportional to the applied unbalanced force in the direction of force".

Mathematical Expression:

Let Mass of an object = m kg

Initial velocity of an object = u m/s

Final velocity of an object = v m/s

Initial momentum $p_1 = mu$

Final momentum $p_2 = mv$

Change in momentum $= P_2 - P_1$

$= mv - mu$

$= m(v-u)$

Rate of change of momentum $= \frac{m(v-u)}{t}$

According to 2^{nd} law, rate of change of momentum $\propto$ force

$\therefore F \propto \frac{m(v-u)}{t}$

But $\frac{v-u}{t} = a$ (from 1st equ) $\quad \therefore F \propto ma$

So, $F = k\,ma$

Unit of force is so chosen so that $k = 1$ $\quad \therefore F = ma$

$F = k \times m \times a$

1 unit of force $= k \times (1\ kg) \times (1\ ms^{-2})$

One unit of force is defined as the amount that produces an acceleration of $1\ m/s^2$ in an object of 1 kg mass.

Thus the value of k become 1

Therefore $F = ma$.

Unit of force is $kg\ ms^{-2}$ or Newton.

Define 1 Newton: When an acceleration of $1\ m/s^2$ is seen in a body of mass 1 kg, then the force applied on the body is said to be 1 Newton.

The second law motion gives us a method to measure the force acting on an object as a product of its mass and acceleration.

Second law of motion in everyday life.

(a) While catching a fast moving cricket ball, a fielder in the ground gradually pulls his hand backwards with the moving ball. In doing so, the fielder increases the time during which the high velocity of the moving ball decreases to zero. Thus, the acceleration of ball decreases and impact is reduced.

(b) In a high jump athletic event, the athletic are made to fall either on a cushioned bed or on a sand bed. This is to increase the time of the athlete's fall to stop after making the jump. This decreases the rate of change of momentum and hence the force.

Derivation of Newton's first law of motion from 2nd law

$$F = ma \quad \text{(Second law of motion)}$$

$$F = \frac{m(v-u)}{t} \quad \left[\because a = \frac{v-u}{t}\right]$$

That is, when $F = 0$, $v = u$ for whatever time, t is taken. This means that the object will continue moving with uniform velocity, u throughout the time, t.

If u is zero then v will also be zero. That is the object will remain at rest.

Q:1) Explain why some of the leaves get detached from a tree if we vigorously shake its branch.

ANS) When the branch of a tree is shaken vigorously, it comes in motion while the leaves tend to remain at rest due to inertia of rest. Therefore, some leaves get detached from the branch of the tree.

Q.2) Why do you fall in the forward direction when a moving bus brakes to a stop and fall backwards when it accelerates from rest?

ANS) In a running bus, our speed is equal to the speed of the bus. As the moving bus brakes to stop, the lower part of the body being in contact with the bus comes to rest, but upper part due to inertia of motion, remain in the state of motion. Hence, we fall in the forward direction. When the bus accelerates from rest, the feet comes into motion while upper part of the body remains at rest due to inertia of rest, hence, we fall back.

Q.3) An object experiences a net zero external unbalanced force. Is it possible for the object to be travelling with a non-zero velocity? If yes, state the conditions that must be placed on the magnitude and direction of the velocity. If no, provide a reason.

Ans) Yes, it is possible for the object to be travelling with a non-zero velocity, if it experiences a net zero external unbalanced force.
This is due to inertia of motion. If body is initially moving with some velocity on a smooth surface, then it will continue to move with same velocity, though the net external force acting on the body is zero.

For example, when we stop pedaling a moving bicycle, the bicycle begins to slow down and finally comes to rest. This is again because of the friction forces acting opposite to the direction of motion. The force of friction opposes the motion of the bicycle. If there were no unbalanced force of friction and no air resistance, a moving bicycle would go on moving for ever.

Q.4) When a carpet is beaten with a stick, dust comes out of it. Explain.
Ans) When the carpet is beaten, it is suddenly set into motion. The dust particles tend to remain at rest due to the inertia of rest, therefore, dust comes out of it.

Q.5) Why is it advised to tie any luggage kept on the roof of a bus with a rope?

Ans) When the bus starts suddenly, the lower part of luggage kept on the roof being in contact with the bus begin to move forward with the speed of the bus, but the upper part due to inertia of rest tends to remain at rest. Therefore, the upper part is left behind and hence luggage falls backward. So, it is advised to tie any luggage kept on the roof of the bus with a rope.

Q.6) A constant force acts on an object of mass 5 kg for a duration of 2 sec. It increases the objects velocity from 3 m/s to 7 m/s. Find the magnitude of the applied force. Now, if the force was applied for a duration of 5 s, what would be the final velocity of the object?

Solution) $m = 5$ kg $t = 2$ sec $u = 3$ m/s $v = 7$ m/s.

According to Newton 2nd law of motion $F = \frac{m(v-u)}{t}$

$$F = \frac{5(7-3)}{2} = \frac{5 \times 4}{2} = 5 \times 2 = 10 \text{ N}$$

$$\therefore F = \underline{10 \text{ N}}$$

Now,

$F = 10$ N $u = 3$ m/s $t = 5$ sec, $m = 5$ g, $v = ?$

$$F = \frac{m(v-u)}{t}, \quad \frac{5(v-3)}{5} = 10, \quad \underline{v = 13 \text{ m/s}}$$

Q.7) Which would require a greater force – accelerating a 2 kg mass at 5 m/s^2 or 4 kg mass at 2 m/s^2?

Ans)

Using Newton's 2nd law of motion $F = ma$

$m_1 = 2$ kg $a_1 = 5$ m/s^2 $m_2 = 4$ kg $a_2 = 2$ m/s^2

$F_1 = m_1 a_1 = 2 \times 5 = 10$ N $F_2 = m_2 a_2 = 4 \times 2 = 8$ N

$\therefore F_1 > F_2$.

Thus, accelerating a 2 kg mass at 5 m/s^2 would require a greater force.

Q.8) A motor car is moving with a velocity of 108 km/h and it takes 4 s to stop after the brakes are applied. Calculate the force exerted by the brakes on the motorcar if its mass along with the passenger is 1000 kg.

Solution) $u = 108$ km/h $= 108 \times \frac{5}{18}$ m/s $= 30$ m/s

$m = 1000$ kg $t = 4$ s $v = 0$

$$F = \frac{m(v-u)}{t} = \frac{1000\ (0-30)}{4} = \frac{-1000 \times 30}{4} = -7500 \text{ N}.$$

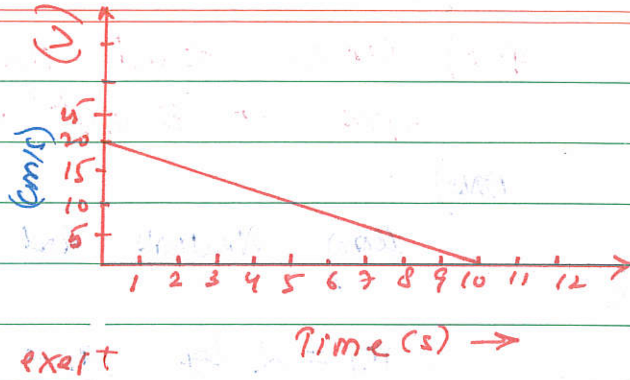
Q.9) A force of 5 N gives a mass m_1 an acceleration of 10 m/s^2 and mass m_2 an acceleration of 20 m/s^2. What acceleration would it given if both the masses were tied together?

Soln) $m_1 = \frac{F}{a_1} = \frac{5}{10} = 0.5$ kg $m_2 = \frac{F_2}{a_2} = \frac{5}{20} = 0.25$ kg.

Total mass $m = m_1 + m_2 = 0.50 + 0.25 = 0.75$ kg.

$a = \frac{F}{m} = \frac{5}{0.75} = 6.67$ m/s^2

10) The velocity time graph of a ball of mass 20 g moving along a straight line on a long table is given



How much force does the table exert on the ball to bring it to rest?

Sol^n) $u = 20$ cm/s $\quad v = 0 \quad t = 10$ sec

$$a = \frac{v-u}{t} = \frac{0-20}{10} = -2 \text{ cm/s}^2 = -0.02 \text{ m/s}^2$$

The force exerted on the ball $F = ma$

$$F = 0.02 \times (-0.02)$$
$$= -0.0004 \text{ N}$$

The negative sign implies that the frictional force exerted by the table is opposite to the direction of motion of the ball.

11) A truck starts from rest and rolls down a hill with a constant acceleration. It travels a distance of 400 m in 20 s. Find its acceleration. Find the force acting on it if its mass is 7 metric tonnes. (Hint: 1 metric tonne = 1000 kg).

Sol^n) $s = 400$ m $\quad t = 20$ s $\quad u = 0$

$s = ut + \frac{1}{2}at^2 \quad 400 = 0 + \frac{1}{2} a \times 20 \times 20 \quad a = 2 \text{ m/s}^2$

$F = ma = 7 \times 1000 \times 2$

$= 14,000$ N

12) A stone of 1 kg is thrown with a velocity of 20 m/s across the frozen surface of a lake and comes to rest after travelling a distance of 50 m. What is the force of friction between the stone and the ice?

Soln) $m = 1$ kg. $u = 20$ m/s $v = 0$ $S = 50$ m

$v^2 - u^2 = 2as$ $0 - 20\times20 = 2a \times 50$

$a = \frac{-20\times20}{2\times50} = -4 \ m/s^2$

$F = ma = (1) \times (-4) = -4 N$

(Negative sign indicates that force is acting opposite to the direction of motion).

13) A 8000 kg engine pulls a train of 5 wagons, each of 2000 kg, along a horizontal track. If the engine exerts a force of 40000 N and the track offers a friction force of 5000 N, the calculate: (a) the net accelerating force (b) the acceleration of the train (c) the force of wagon 1 on wagon 2.

Soln) (a) Force exerted by engine $= 40000$ N

Frictional force $= 5000$ N

The net accelerating force $= 40000 - 5000$ N

$= 35,000$ N

(b) Mass of train $m = 8000 + 2\times5000 = 18,000$ kg

$a = \frac{F}{m} = 35000/18000 = \frac{35}{18} = 1.94 \ m/s^2$

(c) The force exerted of wagon 1 on wagon 2

$=$ Mass of four wagons behind wagon 2 $\times a$

$= 4\times2000\times1.94 = 15,540$ N.

14) An automobile vehicle has a mass of 15000 kg. What must be the force between the vehicle and road if the vehicle is to be stopped with a negative acceleration of 1.7 m/s^2.

Sol^n) $m = 1500$ kg $a = -1.7\ m/s^2$

$f = ma = 1500 \times (1.7)$

$= -2,550$ N

(Negative sign indicate that the force is in a direction opposite to the motion of the vehicle.)

15) Using a horizontal force of 200 N, we intend to move a wooden cabinet across a floor at a constant velocity. What is the friction force that will be exerted on the cabinet?

Sol^n> For a wooden cabinet to move with constant velocity, the net force on the cabinet must be zero.

So, applied force + frictional force = zero.

frictional force + 200 = 0

∴ frictional force = −200 N

Negative sign shows that the frictional force is opposite to the applied force. Therefore, the frictional force exerted on the cabinet must be 200 N.

Third Law of Motion

The first two laws of motion tell us how an applied force changes the motion and provide us with a method of determining the force.

The third law of motion states that "When one object exerts a force on another object, the second object instantaneously exerts a force back on the first."

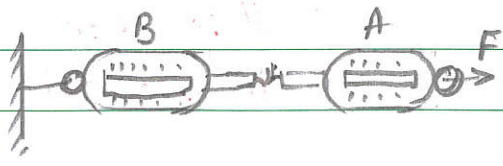
These two forces are always equal in magnitude but opposite in direction.

$$\vec{F_{12}} = -\vec{F_{21}}$$

To every action there is an equal and opposite reaction, but the action and reaction forces act on different bodies.

(a) Spring Balances.

Consider two spring balances A and B. When a force is applied through the free end of spring balances A, it is observed that both spring shows same reading.



It means that the force exerted by spring balance A on balance B is equal but opposite in direction to the force exerted by the balance B on balance A.

(b) Walking : While walking, the road exerts an equal and opposite reaction force on our feet to make us move forward.

(c) Gun : When a gun is fired, it exerts a forward force on the bullet. The bullet exerts an equal and opposite reaction force on gun. This result in the recoil of the gun.

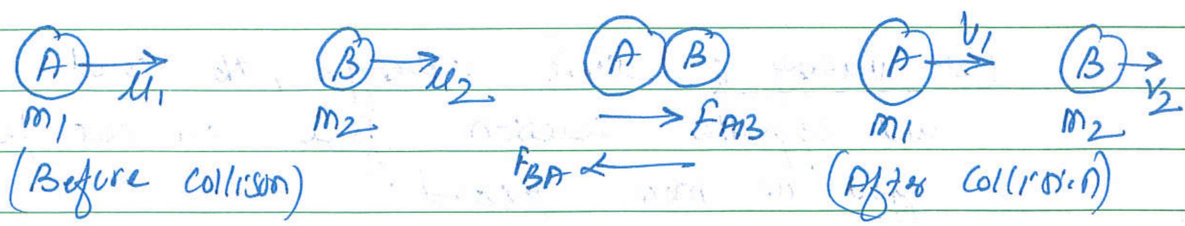
(d) Boat : The third law of motion can also be illustrated when a sailor jumps out of a rowing boat. As the sailor jumps forward, the force on the boat moves it backwards.

Conservation of Momentum

Law of conservation of linear momentum is a extremely important consequence of Newton's third law of motion in combination with the second law of motion.

According to the law of conservation of momentum :
"When two or more bodies acts upon each other, their total momentum remain constant provided no external forces are acting".

Suppose, two objects A and B each of mass m_1 and m_2 are moving initially with velocities u_1 and u_2, strike each other after time t and start moving with velocities v_1 and v_2 respectively.



Initial momentum of object A = m_1u_1

Initial momentum of object B = m_2u_2

Final momentum of object A = m_1v_1

Final momentum of object B = m_2v_2

Rate of change of momentum in A, $f_1 = \frac{m_1v_1 - m_1u_1}{t}$

$$f_1 = \frac{m_1(v_1 - u_1)}{t} \quad \dots (i)$$

Rate of change of momentum in B, $f_2 = \frac{m_2v_2 - m_2u_2}{t}$

$$= \frac{m_2(v_2 - u_2)}{t} \quad \text{---}(ii)$$

We know from 3rd law of motion $f_1 = -f_2$

$$\therefore \frac{m_1(v_1 - u_1)}{t} = -\frac{m_2(v_2 - u_2)}{t}$$

$$m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$

$$\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

Initial Momentum (Before Collision) = Final momentum (After Collision)

Thus, the total momentum of the colliding bodies before collision is equal to the total momentum after collision.

Numericals

① A bullet of mass 20 kg is horizontally fired with a velocity 150 m/s from a pistol of mass 2 kg. What is the recoil velocity of the pistol?

Sol^n> Mass of bullet (m_1) = 20 g = 0·02 kg
Mass of pistol (m_2) = 2 kg

Initially bullet is inside the gun and it is not moving.
Mass = $(m_1 + m_2)$ = (0·02 + 2) = 2·02 kg
$u_1 = 0$

Initial momentum = 2·02 × 0
= 0 ——— (i)

Let the final velocity of pistol be v_2.
and bullet v_1 = 150 m/s

Final momentum = $m_1 v_1 + m_2 v_2$
= 0·02 × 150 + 2 v_2 ——— (ii)

According to the law of conservation of mass
Initial momentum = final momentum

$0 = 0{\cdot}02 \times 150 + 2v_2$
$3 + 2v_2 = 0$
$v_2 = -1{\cdot}5$ m/s.

(−)ve sign indicates that gun recoils in direction opposite to that of the bullet.

② A girl of mass 40 kg jumps with a horizontal velocity of 5 m/s on to a stationary cart with frictionless wheels. The mass of cart is 3 kg. What is her velocity as the cart starts moving? Assume that there is no external unbalanced force working in the horizontal direction.

Sol^n) Let the velocity of girl on the cart, as the cart starts moving be v.

The total momenta of the girl and cart before interaction.

Initial momentum = $40 \times 5 + 3 \times 0$
$= 200$ kg m/s ——— (i)

Total momentum after collision = $(40+3) \times v$
Final momentum = $43v$ kg m/s ——— (ii)

According to the law of conservation of momentum from (i) & (ii)

Initial momentum = Final momentum

$43v = 200$

$v = \frac{200}{43} = 4.65$ m/s.

The girl on cart would move with a velocity of 4.65 m/s in the direction in which girl jumped.

③ Two hockey players viz A of mass 50 kg is moving with a velocity of 4 m/s and another one B belonging to opposite team with mass 60 kg is moving with 3 m/s, get entangled while chasing a fall down. Find the velocity with which they fall down and in which direction?

Soln) $m_A = 50$ kg $\quad u_A = 4$ m/s

$m_B = 60$ kg $\quad u_B = 3$ m/s

Initial momentum $= m_A u_A + m_B u_B$

$= 50 \times 4 + 60 \times 3$

$= 200 + 180$

$= 380$ kg m/s ——— (i)

Let v be the velocity of the two entangled players after collision,

Final momentum $= (m_A + m_B) v$

$= (50 + 60) v$

$= 110 v$ ——— (ii)

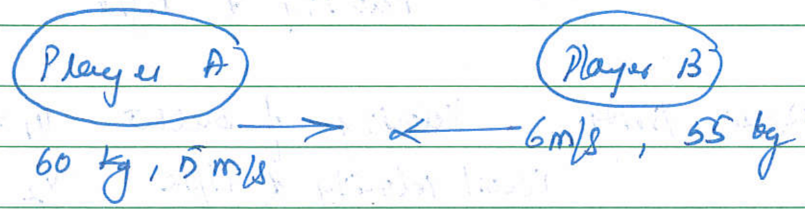
From (i) & (ii) using the law of conservation of momentum $110v = 380$

$v = \frac{380}{110}$

$v = 3.454$ m/s

(4) Two hockey players of opposite teams, while trying to hit a hockey ball on the ground collide and immediately become entangled. One has a mass of 60 kg and was moving with a velocity of 5 m/s while the other has a mass of 55 kg and was moving faster with a velocity 6 m/s towards the first player. In which direction and with what velocity will they move after they become entangled?

Soln>



$m_A = 60$ kg $\quad u_A = 5$ m/s $\qquad m_B = 55$ kg $\quad u_B = -6$ m/s

Total momentum of two players before collision

$$= m_A u_A + m_B u_B$$
$$= 60 \times 5 + 55 \times (-6)$$
$$= 300 - 330$$
$$= -30 \text{ kg m/s} \quad \text{——— (i)}$$

Let V is the velocity of the two entangled players after the collision, then total momentum

$$= (m_A + m_B) V$$
$$= (60 + 55) V$$
$$= 115 V \quad \text{——— (ii)}$$

From (i) & (ii) using the law of conservation of momentum

$$115 V = -30$$
$$V = \frac{-30}{115} = -0.26 \text{ m/s}$$

Thus, the two entangled players would move with velocity 0.26 m/s from right to left.

(5) From a rifle of mass 4 kg, a bullet of mass 50 g is fired with an initial velocity of 35 m/s. Calculate the initial recoil velocity of the rifle.

Soln) Mass of bullet $m_1 = 50\ g = 0.05\ kg$
Mass of rifle $m_2 = 4\ kg$

Before firing, velocity of bullet $u_1 = 0$
velocity of rifle $u_2 = 0$

After firing, velocity of bullet $v_1 = 35\ m/s$
recoil velocity of rifle $v_2 = ?$

Total momentum of the bullet and rifle before the fire, when the rifle is at rest is zero.

$$\therefore m_1u_1 + m_2u_2 = 0 \quad \text{——— (i)}$$

Total momentum of the bullet and rifle after firing

$$= m_1v_1 + m_2v_2$$
$$= 0.05 \times 35 + 4 \times v_2$$
$$= 1.75 + 4v_2 \quad \text{——— (ii)}$$

As there is no external force, so according to the law of conservation of momentum,

Total momentum before fire = Total momentum after fire

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$1.75 + 4v_2 = 0$$
$$v_2 = \frac{-1.75}{4} = -0.44\ m/s$$

Negative sign indicates that the direction in which the pistol would recoil is opposite to that of bullet.

(6) Two object of masses 100 g and 200 g are moving along the same line and direction with velocities of 2 m/s and 1 m/s respectively. They collide and after the collision the first object moves at a velocity of 1.67 m/s. Determine the velocity of the second object.

Soln > $m_1 = 100\ g = 0.1\ kg$ $m_2 = 200\ g = 0.2\ kg$.

$u_1 = 2\ m/s$ $u_2 = 1\ m/s$

$V_1 = 1.67\ m/s$ $V_2 = ?$

According to the law of conservation of momentum,

Total momentum before collision = Total momentum after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$0.1 \times 2 + 0.2 \times 1 = 0.1 \times 1.67 + 0.2 \times V_2$$

$$0.4 = 0.167 + 0.2\, V_2$$

$$0.2\, V_2 = 0.4 - 0.167$$

$$0.2\, V_2 = + 0.233$$

$$V_2 = \frac{0.233}{0.2} = 1.165\ m/s$$

So the velocity of second object is 1.165 m/s.

(7) Two objects each of mass 1.5 kg are moving in the same straight line but in opposite directions. The velocity of each object is 2.5 m/s before the collision during which they stick together. What will be the velocity of the combined object after collision?

Soln) $m_1 = m_2 = 1.5\ kg$ $u_1 = 2.5\ m/s$ $u_2 = -2.5\ m/s$

$$m_1u_1 + m_2u_2 = (m_1 + m_2)V$$

$$1.5 \times 2.5 + 1.5 \times (-2.5) = 3V, \quad V = 0$$

That is combined object comes to rest after collision.

(8) A hockey ball of mass 200 g travelling at 10 m/s is struck by a hockey stick so as to return it along its original path with a velocity at 5 m/s. Calculate the change of momentum occurred in the motion of the hockey ball by the force applied by the hockey stick.

Soln) Mass of ball $m = 200\ g = 0.2\ kg$.

Initial velocity $u = 10\ m/s$

Final velocity $v = -5\ m/s$

Initial momentum $p_1 = mu$

$= 0.2 \times 10 = 2\ kg\ m/s$

Final momentum $p_2 = mv$

$= 0.2(-5) = -1\ kg\ m/s^2$

Change in momentum $= p_2 - p_1$

$= -1 - 2 = -3\ kg\ m/s$.

That is change in momentum is 3 kg m/s along the return path.

(9) A bullet of mass 10 g travelling horizontally with a velocity of 150 m/s strikes a stationary wooden block and comes to rest in 0.03 sec. Calculate the distance of penetration of the bullet into the block. Also calculate the magnitude of force exerted by the wooden block on the bullet.

Soln. Mass of bullet $m = 10\ g = 0.01\ kg$ $u = 150\ m/s$

$v = 0$ $t = 0.03$ sec. $a = \frac{v-u}{t} = \frac{0-150}{0.03} = -5000\ m/s^2$

$v^2 - u^2 = 2as$, $s = \frac{v^2-u^2}{2a} = \frac{0 - 150 \times 150}{2 \times (-5000)} = \underline{2.25\ m}$

$F = ma = 0.01 \times (-5000) = \underline{50\ N}$

(10). An object of mass 1 kg travelling in a straight line with a velocity of 10 m/s collides with, and sticks to, a stationary wooden block of mass 5 kg. Then they both move off together in the same straight line. Calculate the total momentum just before the impact and just after the impact. Also, calculate the velocity of the combined object.

Sol^n: Mass of object $m_1 = 1$ kg

Velocity of object $u_1 = 10$ m/s

Mass of stationary block $m_2 = 5$ kg

Velocity of block before collision $u_2 = 0$

Total momentum before collision $= m_1u_1 + m_2u_2$

$P_1 = 1 \times 10 + 5 \times 0$

$= 10$ kg m/s ✓

Total combined mass after collision $= m_1 + m_2$

(Since object and block stick together after collision)

Velocity of combined object $= v$

According to the law of conservation of momentum:

Total momentum before collision = Total momentum after collision

$m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$\therefore (1+5)v = 10$

$v = \frac{10}{6} = \frac{5}{3}$ m/s

(11) An object of mass 100 kg is accelerated uniformly from a velocity of 5 m/s to 8 m/s in 6 sec. Calculate the initial and final momentum of the object. Also, find the magnitude of the force exerted on the object.

(Soln):

Mass of object $m = 100$ kg
Initial velocity of object $u = 5$ m/s
Final velocity of object $v = 8$ m/s

acceleration $a = \frac{v-u}{t} = \frac{8-5}{6} = \frac{3}{6} = 0.5\ m/s^2$

$F = ma = 100 \times 0.5 = 50N$

Initial momentum $= mu = 100 \times 5 = 500$ kg m/s
Final momentum $= mv = 8 \times 100 = 800$ kg m/s.

(12) How much momentum will a dumb-bell of mass 10 kg transfer to the floor if it falls from a height of 80 cm? Take its downward acceleration to be $10\ m/s^2$.

(Soln) Mass of dumb bell $m = 10$ kg   Distance covered, $s = 80$ cm
Acceleration $a = 10\ m/s^2$ initial velocity $u = 0$
Let final velocity be v m/s.

Using equation $v^2 - u^2 = 2as$, $v^2 = u^2 + 2as = 2 \times 10 \times 0.8$
$v = 4$ m/s

Hence momentum $= mv = 10 \times 4 = 40$ kg m/s.