MATHEMATICS

Sample Question Paper

4

Section 'A'

Question numbers 1 to 10 carry 1 mark each. For each of these questions four alternative choices have been provided of which only one is correct. Select the correct choice.

1. For some integer q, every odd integer is of the form :
   (a) q
   (b) $q + 1$
   (c) $2q$
   (d) $2q + 1$

2. If the HCF of 65 and 117 is expressible in the form $65\,m - 117$, find the value of m :
   (a) $m = 2$
   (b) $m = 4$
   (c) $m = 3$
   (d) $m = 1$

3. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then the product of the other two zeroes is :
   (a) $b - a + 1$
   (b) $b - a - 1$
   (c) $a - b + 1$
   (d) $a - b - 1$

4. Which of the following equations has two distinct real roots?
   (a) $2x^2 - 3\sqrt{2}x + \dfrac{9}{4} = 0$
   (b) $x^2 + x = -5 = 0$
   (c) $x^2 + 3x + 2\sqrt{2} = 0$
   (d) $5x^2 - 3x + 1 = 0$

5. $AOBC$ is a rectangle whose three vertices are vertices $A\ (0, 3)$, $O\ (0, 0)$ and $B\ (5, 0)$. The length of its diagonal is :
   (a) 5
   (b) 3
   (c) $\sqrt{34}$
   (d) 4

   OR

   The point which lies on the perpendicular bisector of the line segment joining the points A ($-2, -5$) and B ($2, 5$) is :
   (a) $(0, 0)$
   (b) $(0, 2)$
   (c) $(2, 0)$
   (d) $(-2, 0)$

6. It is given that $\triangle ABC \sim \triangle DFE$, $\angle A = 30°$, $\angle C = 50°$, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm. Then, the true statement is :
   (a) $DE = 12$ cm, $\angle F = 50°$
   (b) $DE = 12$ cm, $\angle F = 100°$
   (c) $EF = 12$ cm, $\angle D = 100°$
   (d) $EF = 12$ cm, $\angle D = 30°$

7. In the given figure, if $\angle AOB = 125°$, then $\angle COD$ is equal to :
   (a) $62.5°$
   (b) $45°$
   (c) $35°$
   (d) $55°$
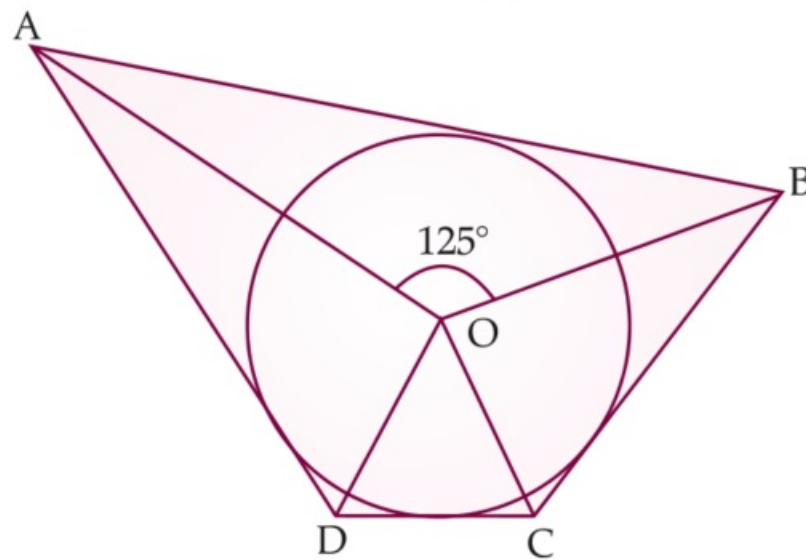
8. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is :
   (a) 31 cm
   (b) 25 cm
   (c) 62 cm
   (d) 50 cm

   OR

   If the perimeter of a circle is equal to that of a square, then the ratio of their areas is :
   (a) 22 : 7
   (b) 14 : 11
   (c) 7 : 22
   (d) 11 : 14

9. If two solid hemispheres of same base radius 'r' are joined together along their bases, then curved surface area of this new solid is :
   (a) $4\pi r^2$
   (b) $6\pi r^2$
   (c) $3\pi r^2$
   (d) $8\pi r^2$

10. An event is very unlikely to happen. Its probability is closest to :

  (a) 0.0001 (b) 0.001 (c) 0.01 (d) 0.1

<div align="center">OR</div>

The probability that a non-leap year selected at random will contain 53 Sundays is :

  (a) $\dfrac{1}{7}$ (b) $\dfrac{2}{7}$ (c) $\dfrac{3}{7}$ (d) $\dfrac{5}{7}$

Question numbers 11 to 20 carry 1 mark each.

11. Explain why 13233343563715 is a composite number ?

12. Write whether rational number $\dfrac{7}{75}$ will have terminating decimal expansion or a non-terminating decimal.

AI 13. Check whether for all values of c, the pair of equations $x - 2y = 8$ and $5x - 10y = c$, have a unique solution or not.

<div align="center">OR</div>

If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then find value of k.

14. For what value of n, are the nth term of two A.Ps, 63, 65, 67 and 3, 10, 17.... equal ?

<div align="center">OR</div>

Find the first four terms of an A.P. whose first term is $3x + y$ and common difference is $x - y$.

15. If the centre and radius of circle is (3, 4) and 7 units respectively, then what is the position of the point $A(5, 8)$ with respect to circle ?

16. What is the maximum number of parallel tangents a circle can have on a diameter ?

17 If A and B are acute angles and $\sin A = \cos B$, then find the value of $A + B$.

18. If the circumferences of two concentric circles forming a ring are 88 cm and 66 cm respectively. Find the width of the ring.

19. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere ?

20. Which central tendency is obtained by the abscissa of point of intersection of less than type and more than type ogives ?
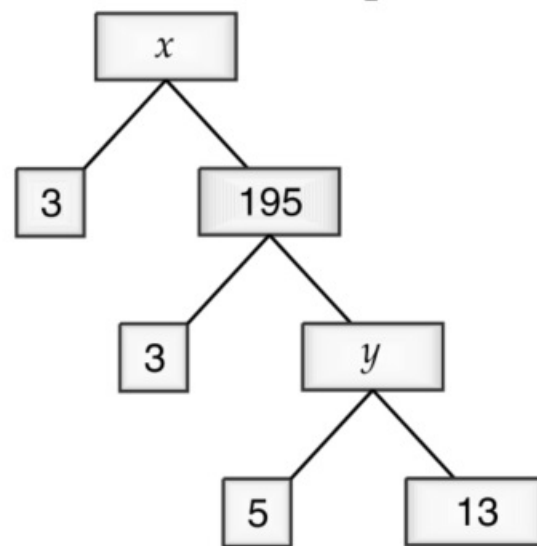
<div align="center">OR</div>

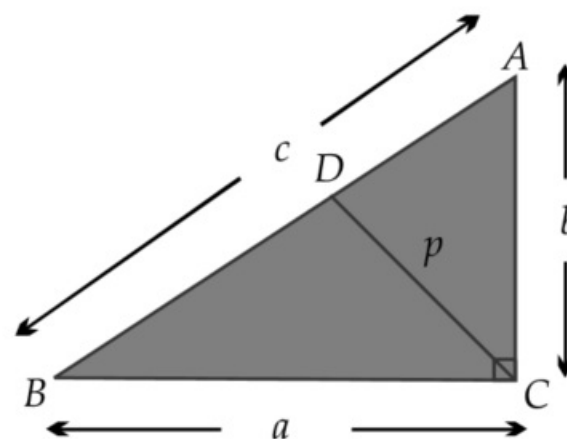Find the median of the data, using an empirical relation when it is given that Mode = 12.4 and Mean = 10.5.

Section 'B'

Question numbers 21 to 26 carry 2 marks each.

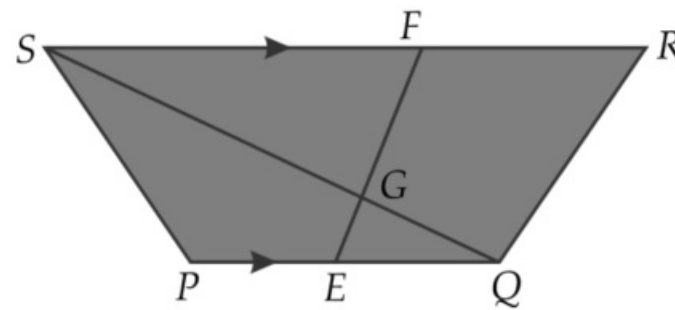**21.** Complete the following factor tree and find the composite number x.

22. ABC is a right triangle, right angled at C. Let $BC = a$, $CA = b$, $AB = c$ and p be the length of perpendicular from C to AB. Prove that $cp = ab$.

OR

In the figure, PQRS is a trapezium in which PQ || RS. On PQ and RS, there are points E and F respectively such that EF intersects SQ at G. Prove that $EQ \times GS = GQ \times FS$.

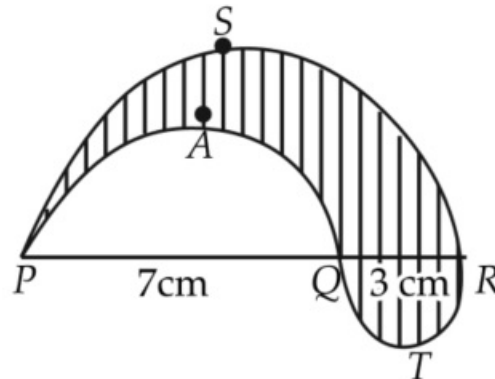23. If $\sin(36 + \theta)° = \cos(16 + \theta)°$, then find θ, where $(36 + \theta)°$ and $(16 + \theta)°$ are both acute angles.

OR

Prove that :

$$\frac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta} = 1$$

24. From the top of light house, 40 m above the water, the angle of depression of a small boat is 60°. Find how far the boat is from the base of the light house.

25. In the fig., *PSR, RTQ* and *PAQ* are three semi-circles of diameters 10 cm, 3 cm and 7 cm respectively. Find the perimeter of shaded region. $\left(\text{Use } \pi = \dfrac{22}{7} \right)$

26. The marks obtained by 100 students in an examination are given below :

Marks	30–35	35–40	40–45	45–50	50–55	55–60	60–65
Number of Students	14	16	28	23	18	8	3

Find the mean.

Section 'C'

Question numbers 27 to 34 carry 3 marks each.

27. Verify whether 2, 3 and $\dfrac{1}{2}$ are the zeroes of the polynomial $p(x) = 2x^3 - 11x^2 + 17x - 6$.

OR

Rehman's mother has given him money to buy some boxes from the market at the rate of $x^2 + 2x - 3$. The total amount of money given by his mother is represented by $4x^4 + 2x^3 - 2x^2 + x - 1$. Out of this money, he donated some amount to a child who was studying in the light of a street lamp. Find how much amount of money he must have so that he is able to buy exact and maximum number of boxes from the market.

28. Determine the values of m and n so that the following system of linear equations have infinite number of solutions :

$(2m - 1)x + 3y - 5 = 0$

and $3x + (n - 1)y - 2 = 0$

29. Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained.

[AI] **30.** If the ratio of the sums of first n terms of two A.P.'s is $(7n + 1) : (4n + 27)$, find the ratio of their 9th terms.

31. If P and Q are the points on side CA and CB, respectively of $\triangle ABC$ right angted at C, Prove that $(AQ^2 + BP^2) = AB^2 + PQ^2$.

32. AB is a chord of circle with centre O. At B, a tangent PB is drawn such that its length is 24 cm. The distance of P from the centre is 26 cm. If the chord AB is 16 cm, find its distance from the centre.
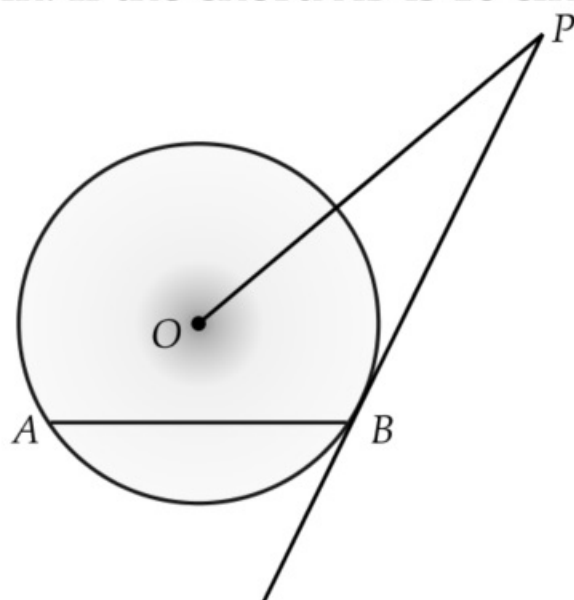
OR

In figure, PQ and RS are two parallel tangents to a circle with center O and another tangent AB will point of contact C intersecting PQ at A and RS at B. Prove that $\angle AOB = 90°$.
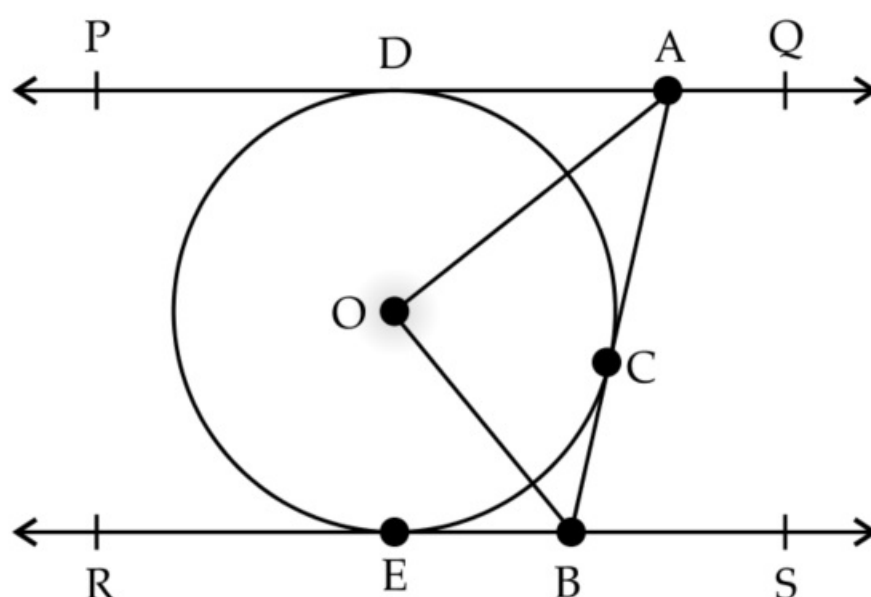
33. If $\sec\theta + \tan\theta = p$, show that $\sec\theta - \tan\theta = \dfrac{1}{p}$. Hence, find the values of $\cos\theta$ and $\tan\theta$.

34. Two different dice are thrown together. Find the probability that the number obtained.
  (i)   have a sum less than 7.
  (ii) have a product less than 16.
  (iii) is a doublet of odd numbers.

OR

One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting (a) Non face card, (b) Black king or a Red queen, (c) Spade card.

Section 'D'

Question numbers 35 to 40 carry 4 marks each.

35. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which tap can fill the tank separately.

36. A(4, – 6), B(3, – 2) and C(5, 2) are the vertices of a ΔABC and AD is its median. Prove that the median AD divides ΔABC into two triangles of equal areas.

OR

If $a \neq b \neq 0$, Prove that the points (a, a^2), (b, b^2), $(0, 0)$ will not be collinear.

37. Construct an isosceles triangle whose base is 6 cm and altitude 4 cm. Then construct another triangle with sides are $\frac{3}{4}$ times the corresponding sides of the isosceles triangle.

38. Two points A and B are on the same side of a tower and in the same straight line with its base. The angle of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 15 m, then find the distance between these points.

OR

An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45º and 60º respectively. Find the width of the river. [Use $\sqrt{3} = 1.732$].

AI 39. A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom of circular ends of the bucket are 20 cm and 12 cm, respectively. Find the height of the bucket and also the area of the metal sheet used in making it. (Use π = 3.14)

OR

In the given figure, ΔABC is a right angled triangle in which ∠A = 90°. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region.
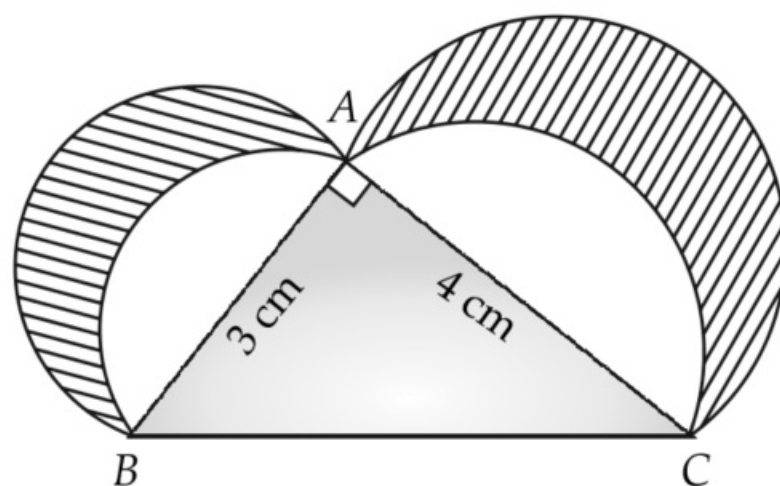
AI 40. If the median of the following frequency distribution is 32.5 find values of f_1 and f_2.

Class	0–10	10–20	20–30	30–40	40–50	50–60	60–70	Total
Frequency	f_1	5	9	12	f_2	3	2	40