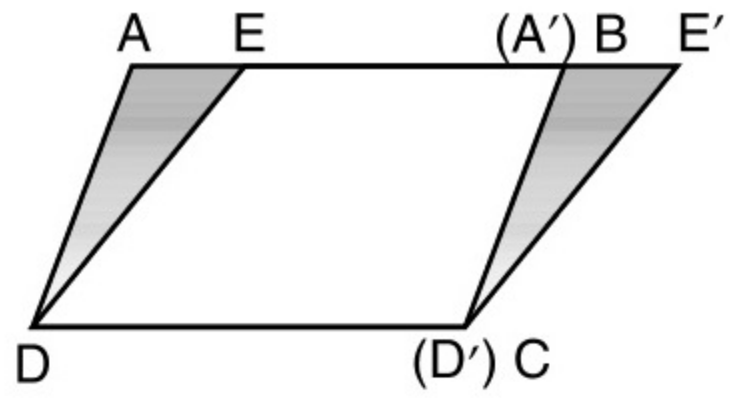
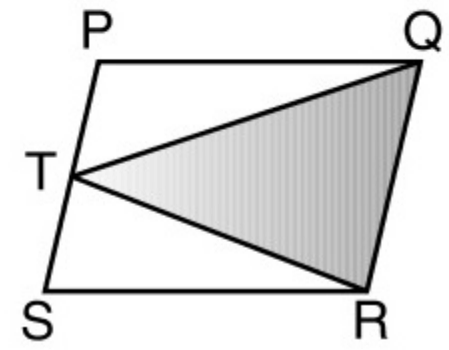
CHAPTER 9 : Area of Parallelograms and Triangles

- Two figures are said to be on the same base and between the same parallels, if they have a common base (side) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base. Examples :





- Area of triangle is half the product of its base (or any side) and the corresponding altitude (or height).
- Two triangles with same base (or equal bases) and equal areas will have equal corresponding altitudes.
- A median of a triangle divides it into two triangles of equal areas.

Theorems and Proofs(wherever required) :

Theorem : 1

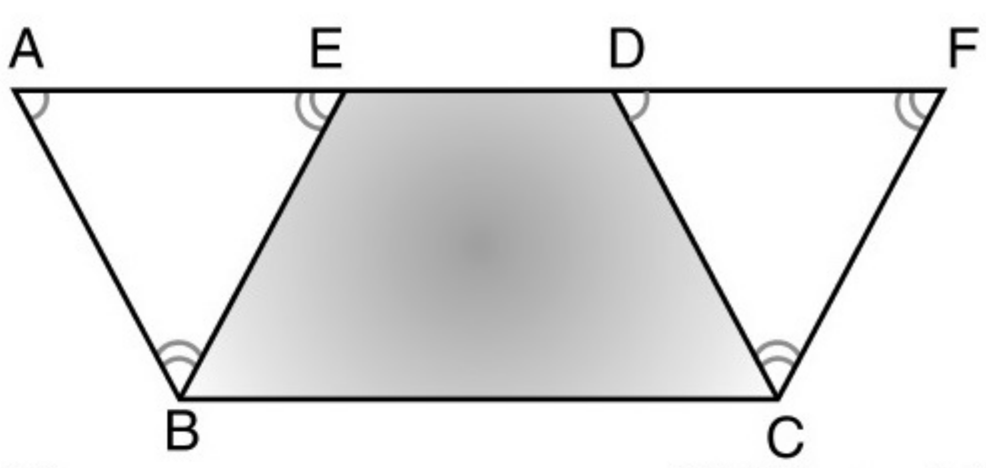
Statement : Parallelograms on the same base and between the same parallels are equal in area.

Given : Two parallelograms $ABCD$ and $EBCF$ on the same base BC and between the same parallels BC and AF.

To prove : $ar\ (||^{gm}\ ABCD) = ar\ (||^{gm}\ EBCF)$

Proof : In ΔABE and ΔDCF,

$$\angle EAB = \angle FDC \quad \left(\begin{array}{l} AB \parallel CD,\ AF \text{ is the transversal} \\ \because \text{pair of corresponding angles are equal} \end{array}\right) \qquad ...(i)$$



$$\angle BEA = \angle CFD \quad \left(\begin{array}{l} AB \parallel CD,\ AF \text{ is the transversal} \\ \because \text{pair of corresponding angles are equal} \end{array}\right) \qquad ...(ii)$$

Now, $\angle EAB + \angle ABE + \angle BEA = \angle FDC + \angle DCF + \angle CFD = 180^\circ$

(sum of measure of the interior angles of a triangle is 180°)

$\therefore$ $\angle ABE = \angle DCF$ ...(iii)[using (i) and (ii)]

Also, $AB = DC$ (Opposite side of $||^{gm}$ are equal) ...(iv)

Therefore, using (i), (iii) and (iv), $\Delta ABE \cong \Delta DCF$ (*ASA* rule)

$\therefore$ $ar\ (\Delta ABE) = ar\ (\Delta DCF)$

$\therefore$ $ar\ (||^{gm}\ ABCD) = ar\ (\Delta ABE) + ar$ (trapezium $EBCD$)

$= ar\ (\Delta DCF) + ar$ (trapezium $EBCD$)

$= ar\ (||^{gm}\ EBCF)$

Hence, $ar\ (||^{gm}\ ABCD) = ar\ (||^{gm}\ EBCF)$

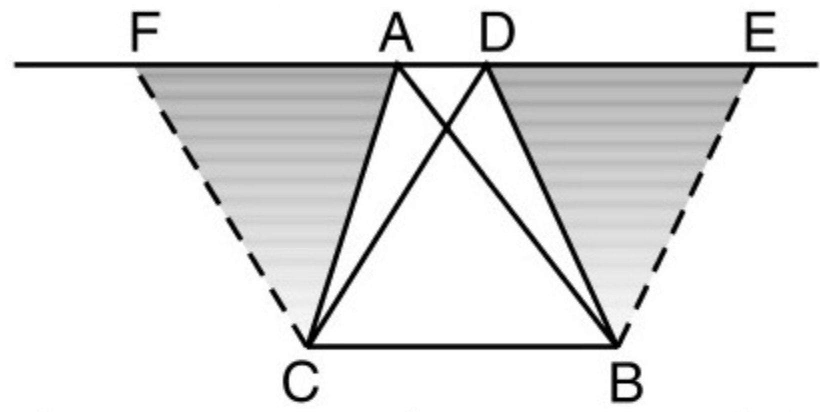
Theorem : 2

Statement : Two triangles on the same base (or equal bases) and between the same parallels are equal in area.

Given : Two triangles ΔABC and ΔDBC are on the same base BC and between the same parallels EF and BC.

To prove : $ar\ (\Delta ABC) = ar\ (\Delta DBC)$

Construction : Through B, draw $BE \parallel AC$, intersecting the line AD produced in E and through C, draw $CF \parallel BD$, intersecting the line AD produced in F.



Proof : $EACB$ and $DFCB$ are parallelograms (since two pairs of opposite sides are parallel).

Also $\parallel^{gm}$ $EACB$ and $\parallel^{gm}$ $DFCB$ are on the same base BC and between the same parallels EF and BC.

$$ar\ (\parallel^{gm} EACB) = ar\ (\parallel^{gm} DFCB) \quad \text{...(i)}$$

Now, AB is the diagonal of $\parallel^{gm}$ $EACB$

$\therefore$ $ar\ (\Delta EAB) = ar\ (\Delta ABC)$ [Diagonal of parallelogram divides it in congruent triangle]

$$\therefore \quad ar\ (\Delta ABC) = \frac{1}{2}\ ar\ (\parallel^{gm} EACB) \quad \text{...(ii)}$$

Similarly, $$ar\ (\Delta DBC) = \frac{1}{2}\ ar\ (\parallel^{gm} DFCB) \quad \text{...(iii)}$$

From equations (i), (ii) and (iii), we get, $ar\ (\Delta ABC) = ar\ (\Delta DBC)$

Theorem : 3

Statement : Two triangles on the same base (or equal bases) and equal areas lie between the same parallels.